中國科學院物理研究所

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T-linear resistivity and superconductivity

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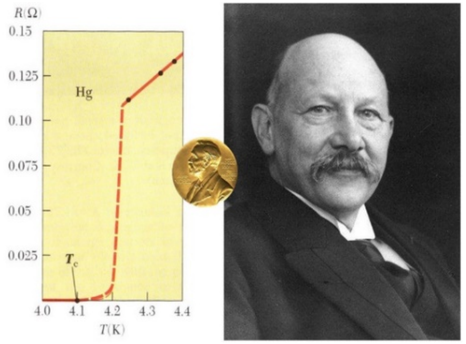
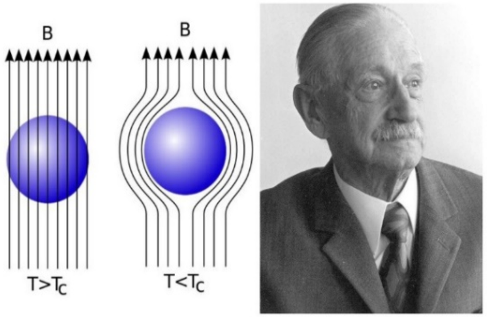
outline

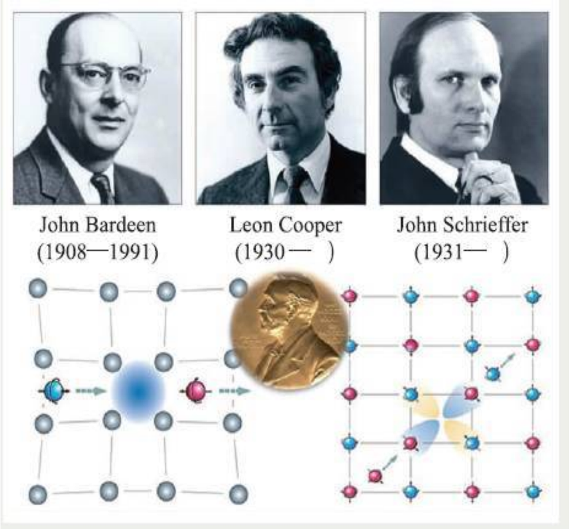
I background



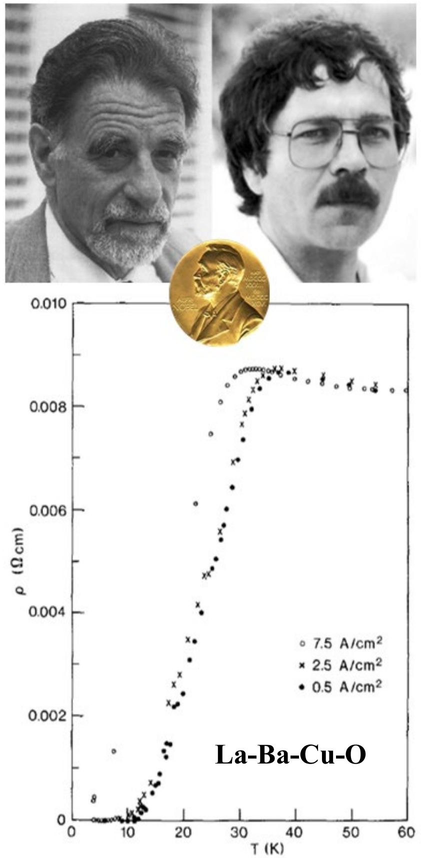
In 1911, zore resistivity of Hg [1]



In 1933, Messina effect

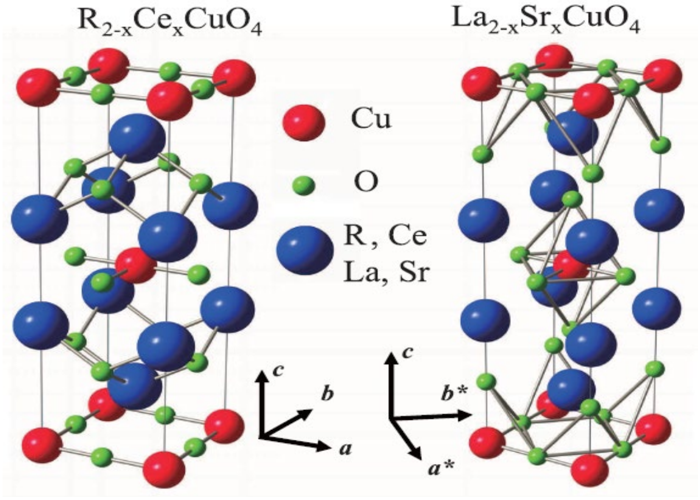


In 1957, BSC thoery[2]

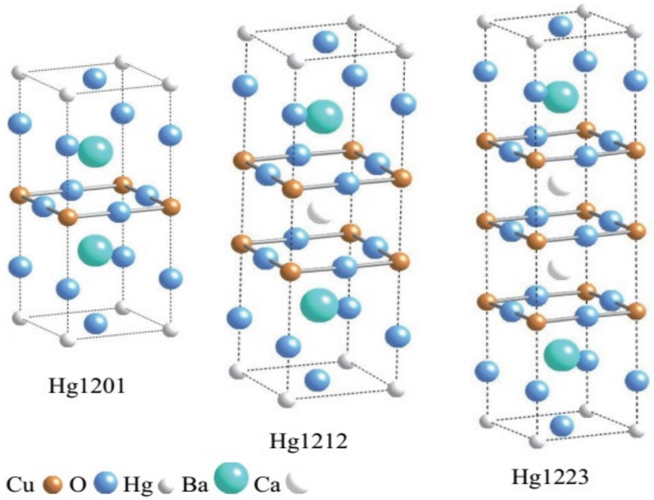


In 1986, cuprate superconductor

Crystal structure of cuprate superconductor

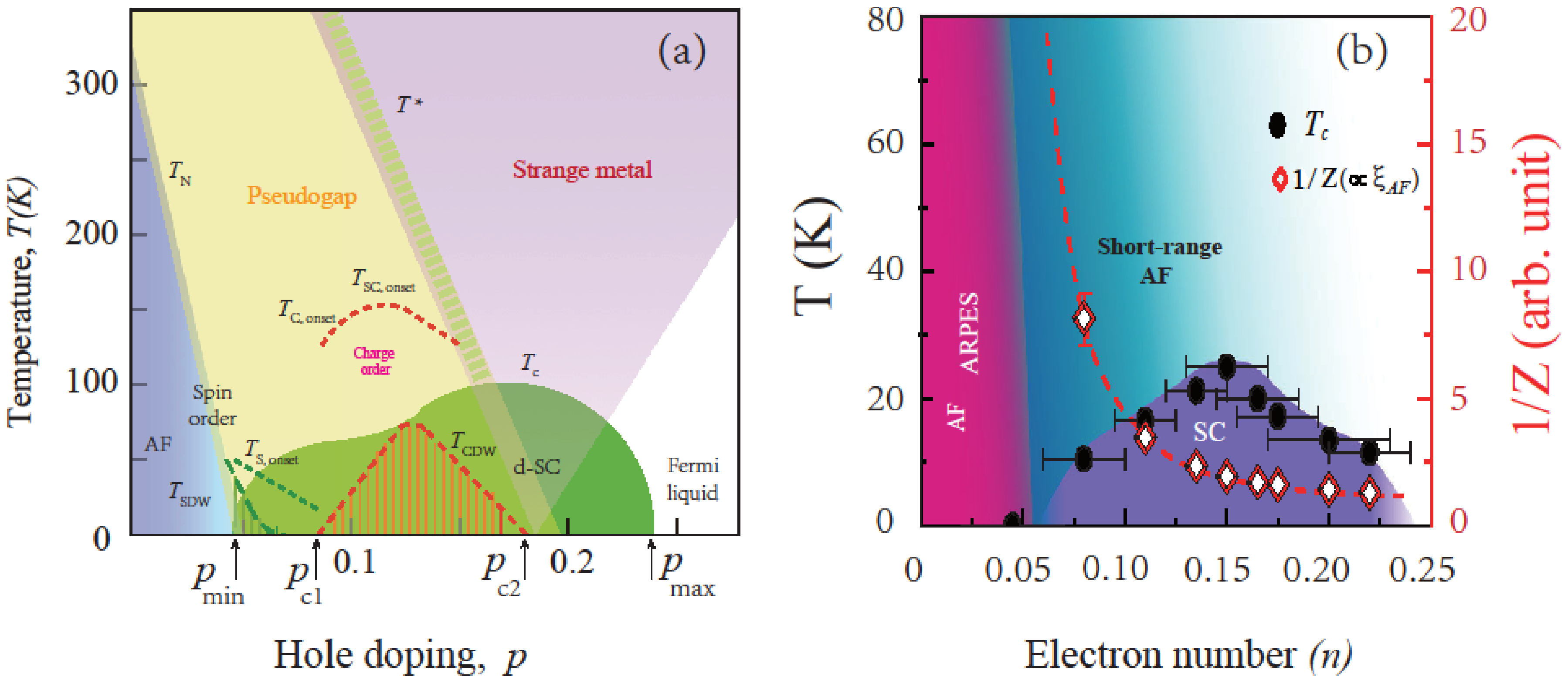


Lift figure is the structure of ele-doping $R_{2-x}Ce_xCuO_4$, the figure on the right is the structure of hole-doped $La_{2-x}Sr_xCuO_4$.



The crystal structure of Hg-based cuprate: (a) Hg1201 (T_c=94K), (b) Hg1212 (T_c=127K), (c) Hg1223 (T_c=135K)

T-doping phase map of cuprate superconductor



空穴掺杂型(a图)和电子掺杂型(b图) 铜氧化物超导体随着温度和掺杂浓度变化的相图[3]

Ⅱ methods

① Many-body physics and Green function

② Kinetic energy driven superconductivity

③ Fermion-spin theory

④ Full charge-spin recombination

Model of strong correlation electron

t-J model：

$$H = -t \sum_{l\hat{\eta}\sigma} C^\dagger_{l\sigma} C_{l+\hat{\eta}\sigma} + t' \sum_{l\hat{\tau}\sigma} C^\dagger_{l\sigma} C_{l+\hat{\tau}\sigma} + \mu \sum_{l\sigma} C^\dagger_{l\sigma} C_{l\sigma} + J \sum_{l\hat{\eta}} \mathbf{S}_l \cdot \mathbf{S}_{l+\hat{\eta}}$$

Constrain of no couple of electron occupation：

$$\sum_{\sigma} C^\dagger_{i\sigma} C_{i\sigma} \leq 1,$$

Fermion-spin theory

Charge-spin separation：

$$C_{i\uparrow} = h^{\dagger}_{i\uparrow} S^{-}_{i}, \quad C_{i\downarrow} = h^{\dagger}_{i\downarrow} S^{+}_{i},$$

Then, the constrain is satisfied automatically： $\sum_{\sigma} C^{\dagger}_{i\sigma} C_{i\sigma} = 1 - h^{\dagger}_{i} h_{i} \leq 1$

Hamilton in the Fermion-spin representation：

$$\begin{aligned} H &= t \sum_{i\hat{\eta}} (h^{\dagger}_{i+\hat{\eta}\uparrow} h_{i\uparrow} S^{+}_{i} S^{-}_{i+\hat{\eta}} + h^{\dagger}_{i+\hat{\eta}\downarrow} h_{i\downarrow} S^{-}_{i} S^{+}_{i+\hat{\eta}}) \\ &\quad - t' \sum_{i\hat{\tau}} (h^{\dagger}_{i+\hat{\tau}\uparrow} h_{i\uparrow} S^{+}_{l} S^{-}_{i+\hat{\tau}} + h^{\dagger}_{i+\hat{\tau}\downarrow} h_{i\downarrow} S^{-}_{i} S^{+}_{i+\hat{\tau}}) \\ &\quad - \mu_h \sum_{i\sigma} h^{\dagger}_{i\sigma} h_{i\sigma} + J_{\mathrm{eff}} \sum_{i\hat{\eta}} \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{\eta}}, \end{aligned}$$

Hamilton under the mean-field approximate:

Hamilton of charge carriers:

$$H_h = t\chi_1 \sum_{i\hat{\eta}\sigma} h^\dagger_{i+\hat{\eta}\sigma} h_{i\sigma} - t'\chi_2 \sum_{i\hat{\tau}\sigma} h^\dagger_{i+\hat{\tau}\sigma} h_{i\sigma} - \mu_h \sum_{i\sigma} h^\dagger_{i\sigma} h_{i\sigma}$$

Hamilton of spinon:

$$H_J = \frac{1}{2}\epsilon J_{\rm eff} \sum_{i\hat{\eta}} (S^+_i S^-_{i+\hat{\eta}} + S^-_i S^+_{i+\hat{\eta}}) - t'\phi_2 \sum_{i\tau} (S^+_i S^-_{i+\hat{\tau}} + S^-_i S^+_{i+\hat{\tau}}) + J_{\rm eff} \sum_{i\hat{\eta}} S^z_i S^z_{i+\hat{\eta}}$$

Green function

Diagonal and off-diagonal Green function of charge carriers:

$$g(i-j,t-t')=\langle\langle h_{i\sigma}(t);h_{j\sigma}^{\dagger}(t')\rangle\rangle,$$

$$\Im(i-j,t-t')=\langle\langle h_{i\sigma}(t);h_{j\sigma'}(t')\rangle\rangle,$$

$$\Im^{\dagger}(i-j,t-t')=\langle\langle h_{i\sigma}^{\dagger}(t);h_{j\sigma'}^{\dagger}(t')\rangle\rangle$$

Green function of spinon:

$$D(i-j,t-t')=\langle\langle S_{i}^{+}(t);S_{j}^{-}(t')\rangle\rangle,$$

$$D^{z}(i-j,t-t')=\langle\langle S_{i}^{z}(t);S_{j}^{z}(t')\rangle\rangle$$

Mean field Green function:

Mean field Green function of charge carrier:

$$g^{(0)}(\mathbf{k}, i\omega_m) = \frac{1}{i\omega_m - \xi_{\mathbf{k}}}$$

here， the excitation spectrum of charge carrier:

$$\xi_{\mathbf{k}} = Zt\chi_1\gamma_{\mathbf{k}} - Zt'\chi_2\gamma'_{\mathbf{k}} - \mu_h$$

$$\gamma_{\mathbf{k}} = \frac{1}{2}(cosk_x + cosk_y)$$

$$\gamma'_{\mathbf{k}} = cosk_xcosk_y$$

Mean field Green function of spinion

$$D^{(0)}(\mathbf{k}, i\omega_m) = \frac{b_{\mathbf{k}}}{i\omega_m^2 - \omega_{\mathbf{k}}^2} = \frac{b_{\mathbf{k}}}{2\omega_{\mathbf{k}}}\left(\frac{1}{i\omega_m - \omega_{\mathbf{k}}} - \frac{1}{i\omega_m + \omega_{\mathbf{k}}}\right),$$

$$D_z^{(0)}(\mathbf{k}, i\omega_m) = \frac{b_{\mathbf{k}}^z}{i\omega_m^2 - \omega_{\mathbf{k}}^{z\,2}} = \frac{b_{\mathbf{k}}^z}{2\omega_{\mathbf{k}}^z}\left(\frac{1}{i\omega_m - \omega_{\mathbf{k}}^z} - \frac{1}{i\omega_m + \omega_{\mathbf{k}}^z}\right)$$

spin excitation spectrum:

$$\omega_{\mathbf{k}}^2 = A_1\gamma_{\mathbf{k}}^2 + A_2\gamma_{\mathbf{k}}'^2 + A_3\gamma_{\mathbf{k}}\gamma_{\mathbf{k}}' + A_4\gamma_{\mathbf{k}} + A_5\gamma_{\mathbf{k}} + A_6,$$

$$\omega_{\mathbf{k}}^z = A_1^z\gamma_{\mathbf{k}}^2 + A_2^z\gamma_{\mathbf{k}}\gamma_{\mathbf{k}}' + A_3^z\gamma_{\mathbf{k}} + A_4^z\gamma_{\mathbf{k}}' + A_5^z,$$

the weight of spin excitation spectrum:

$$b_{\mathbf{k}} = \lambda_1(2\epsilon\chi_1^z + \chi_1)\gamma_{\mathbf{k}} - 2\lambda_2\chi_2^z\gamma_{\mathbf{k}}' - \lambda_1(\epsilon\chi_1 + 2\chi_1^z) + \lambda_2\chi_2$$

$$b_{\mathbf{k}}^z = \lambda_1\epsilon\chi_1\gamma_{\mathbf{k}} - \lambda_2\chi_2\gamma_{\mathbf{k}}' - \lambda_1\epsilon\chi_1 + \lambda_2\chi_2,$$

some constant defined：

$$A_1 = -\epsilon\lambda_1^2[\alpha\chi_1^z + \frac{1}{2}\alpha\chi_1], \quad A_2 = \alpha\lambda_2^2\chi_2^z, \quad A_3 = -\alpha\lambda_1\lambda_2[\epsilon\chi_1^z + \frac{1}{2}\chi_1 + \epsilon\chi_1^z]$$

$$A_4 = -\epsilon\lambda_1^2[\frac{3}{8}\epsilon\alpha\chi_1 + \frac{3}{4}\alpha\chi_1^z + \frac{1}{8}(1-\alpha) + \frac{1}{2}c_1\alpha + \alpha c_1^z] + \frac{1}{2}\alpha\lambda_1\lambda 2(c_3 + \epsilon\chi_2)$$

$$A_5 = \alpha\lambda_1\lambda_2(\frac{1}{2}\epsilon\chi_1 + \chi_1^z + c_3^z) - \frac{3}{8}\alpha\lambda_2^2\chi_2$$

$$A_6 = \lambda_1^2[\alpha\epsilon^2(\frac{1}{2}c_1 - \frac{1}{Z}\chi_1^z) + \frac{1}{4Z}(1-\alpha)(1+\epsilon^2) + \alpha c_1^z - \frac{1}{2Z}\epsilon\alpha\chi_1]$$
$$+\lambda_2^2[\frac{1}{2}\alpha c_2 + \frac{1}{4Z}(1-\alpha) - \frac{1}{Z}\alpha\chi_2^z] - \epsilon\lambda_1\lambda_2\alpha c_3$$

$$A_1^z = \alpha\epsilon\chi_1\lambda_1^2, \quad A_2^z = -\alpha\chi_2\lambda_1\lambda_2$$

$$A_3^z = -\frac{3}{4}\lambda_1^2\alpha\epsilon\chi_1 - \lambda_2^2[\alpha c_1 + \frac{1}{8}(1-\alpha)] + \lambda_1\lambda_2\alpha(\epsilon c_3 - \chi_2)$$

$$A_4^z = -\lambda_2^2[\alpha c_2 + \frac{1}{8}(1-\alpha)] + \lambda_1\lambda_2\alpha\epsilon c_3$$

$$A_5^z = \lambda_1^2\epsilon\{\epsilon[\alpha c_1 + \frac{1}{8(1-\alpha)}] - \frac{1}{4}\alpha\chi_1\} + \lambda_2^2[\alpha c_2 + \frac{1}{8}(1-\alpha)] - 2\lambda_1\lambda_2\alpha\epsilon c_3$$

Dyson equations of charge carriers:

$$g(\mathbf{k}, i\omega_m) = g^{(0)}(\mathbf{k}, i\omega_m) + g^{(0)}(\mathbf{k}, i\omega_m)[\Sigma_{\rm ph}^{(h)}(\mathbf{k}, i\omega_m)g(\mathbf{k}, i\omega_m) - \Sigma_{\rm pp}^{(h)}(-\mathbf{k}, -i\omega_m)\Im^{\dagger}(-\mathbf{k}, -i\omega_m)],$$

$$\Im^{\dagger}(\mathbf{k}, i\omega_m) = g^{(0)}(-\mathbf{k}, -i\omega_m)[\Sigma_{\rm ph}^{(h)}(-\mathbf{k}, -i\omega_m)g(\mathbf{k}, i\omega_m) - \Sigma_{\rm pp}^{(h)}(-\mathbf{k}, -i\omega_m)\Im^{\dagger}(-\mathbf{k}, -i\omega_m)]$$

Here, the normal and abnormal self energy of charge carrier:

$$\begin{aligned}\Sigma_{\rm ph}^{(h)}(\mathbf{k}, i\omega_m) &= \frac{1}{N^2}\sum_{\mathbf{k}_1,\mathbf{k}_2}\Lambda^2_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}}\frac{1}{\beta^2}\sum_{i\omega_{m1},i\omega_{m2}} g(\mathbf{k}+\mathbf{k}_1, i\omega_m + i\omega_{m1})D^{(0)}(\mathbf{k}_1, i\omega_{m1}) \\ &\times D^{(0)}(\mathbf{k}_1+\mathbf{k}_2, i\omega_{m1}+i\omega_{m2}) \\ \Sigma_{\rm pp}^{(h)}(\mathbf{k}, i\omega_m) &= \frac{1}{N^2}\sum_{\mathbf{k}_1,\mathbf{k}_2}\Lambda^2_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}}\frac{1}{\beta^2}\sum_{i\omega_{m1},i\omega_{m2}} \Im^{\dagger}(-\mathbf{k}-\mathbf{k}_1, -i\omega_m - i\omega_{m1})D^{(0)}(\mathbf{k}_1, i\omega_{m1}) \\ &\times D^{(0)}(\mathbf{k}_1+\mathbf{k}_2, i\omega_{m1}+i\omega_{m2})\end{aligned}$$

Diagonal and off-diagonal Green function of charge carrier:

$$g(\mathbf{k}, i\omega_m) = \frac{i\omega_m[1 - \Sigma^{(h)}_{\rm pho}(\mathbf{k}, i\omega_m)] + [\xi_{\mathbf{k}} + \Sigma^{(h)}_{\rm phe}(\mathbf{k}, i\omega_m)]}{\{i\omega_m[1 - \Sigma^{(h)}_{\rm pho}(\mathbf{k}, i\omega_m)]\}^2 - [\xi_{\mathbf{k}} + \Sigma^{(h)}_{\rm phe}(\mathbf{k}, i\omega_m)]^2 - [\Sigma^{(h)}_{\rm pp}(\mathbf{k}, i\omega_m)]^2}$$

$$\Im^{\dagger}(\mathbf{k}, i\omega_m) = -\frac{\Sigma^{(h)}_{\rm pp}(\mathbf{k}, i\omega_m)}{\{i\omega_m[1 - \Sigma^{(h)}_{\rm pho}(\mathbf{k}, i\omega_m)]\}^2 - [\xi_{\mathbf{k}} + \Sigma^{(h)}_{\rm phe}(\mathbf{k}, i\omega_m)]^2 - [\Sigma^{(h)}_{\rm pp}(\mathbf{k}, i\omega_m)]^2}$$

Renormalization factor and gap of charge carrier in long wave limit approximation

$$Z^{-1}_{hF} = 1 - \Sigma^{(h)}_{\rm pho}(\mathbf{k}, 0) = 1 - \Sigma^{(h)}_{\rm pho}(\mathbf{k})|_{\mathbf{k}=(\pi,0)}, \quad \bar{\Delta}_h(\mathbf{k}) = \Sigma^{(h)}_{\rm pp}(\mathbf{k}, 0) = \Sigma^{(h)}_{\rm pp}(\mathbf{k}).$$

So, the Green function could be rewrite as:

$$g(\mathbf{k}, i\omega_m) = Z_{hF}\left(\frac{U^2_{h\mathbf{k}}}{i\omega_m - E_{h\mathbf{k}}} + \frac{V^2_{h\mathbf{k}}}{i\omega_m + E_{h\mathbf{k}}}\right),$$

$$\Im^{\dagger}(\mathbf{k}, i\omega_m) = -Z_{hF}\frac{\bar{\Delta}_{hZ}(\mathbf{k})}{2E_{h\mathbf{k}}}\left(\frac{1}{i\omega_m - E_{h\mathbf{k}}} - \frac{1}{i\omega_m + E_{h\mathbf{k}}}\right).$$

Here,

$$U^2_{h\mathbf{k}} = \frac{1}{2}\left(1 + \frac{\bar{\xi}_{\mathbf{k}}}{E_{h\mathbf{k}}}\right),$$

$$V^2_{h\mathbf{k}} = \frac{1}{2}\left(1 - \frac{\bar{\xi}_{\mathbf{k}}}{E_{h\mathbf{k}}}\right).$$

Then, self energy of charge carrier :

$$\Sigma_{\rm ph}^{(h)}(\mathbf{k}, i\omega_m) = (-1)^{\nu_2+\nu_3}\frac{Z_{hF}}{2N^2}\sum_{\substack{\mathbf{k}_1\mathbf{k}_2\\ \nu_1\nu_2\nu_3}}\Lambda_{\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2}\left(1+\frac{\bar{\xi}_{\mathbf{k}+\mathbf{k}_2}}{E_{h\mathbf{k}+\mathbf{k}_2}^{\nu_1}}\right)\frac{b_{\mathbf{k}_1}b_{\mathbf{k}_1+\mathbf{k}_2}}{4\omega_{\mathbf{k}_1}\omega_{\mathbf{k}_1+\mathbf{k}_2}}$$
$$\times\frac{[1+n_B(\omega_{\mathbf{k}_1}^{\nu_2})]n_B(\omega_{\mathbf{k}_1+\mathbf{k}_2}^{\nu_3})+n_F(E_{h\mathbf{k}+\mathbf{k}_1}^{\nu_1})[n_B(\omega_{\mathbf{k}_1}^{\nu_2})-n_B(\omega_{\mathbf{k}_1+\mathbf{k}_2}^{\nu_3})]}{i\omega_m-E_{h\mathbf{k}+\mathbf{k}_1}^{\nu_1}-\omega_{\mathbf{k}_1}^{\nu_2}+\omega_{\mathbf{k}_1+\mathbf{k}_2}^{\nu_3}}$$

$$\Sigma_{\rm pp}^{(h)}(\mathbf{k}, i\omega_m) = (-1)^{\nu_2+\nu_3+1}\frac{1}{N^2}\sum_{\substack{\mathbf{k}_1\mathbf{k}_2\\ \nu_1\nu_2\nu_3}}\Lambda_{\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2}\frac{\bar{\Delta}_{hZ}(\mathbf{k}+\mathbf{k}_2)}{2E_{h\mathbf{k}+\mathbf{k}_2}^{\nu_1}}\frac{b_{\mathbf{k}_1}b_{\mathbf{k}_1+\mathbf{k}_2}}{4\omega_{\mathbf{k}_1}\omega_{\mathbf{k}_1+\mathbf{k}_2}}$$
$$\times\frac{[1+n_B(\omega_{\mathbf{k}_1}^{\nu_2})]n_B(\omega_{\mathbf{k}_1+\mathbf{k}_2}^{\nu_3})+n_F(E_{h\mathbf{k}+\mathbf{k}_1}^{\nu_1})[n_B(\omega_{\mathbf{k}_1}^{\nu_2})-n_B(\omega_{\mathbf{k}_1+\mathbf{k}_2}^{\nu_3})]}{i\omega_m-E_{h\mathbf{k}+\mathbf{k}_1}^{\nu_1}-\omega_{\mathbf{k}_1}^{\nu_2}+\omega_{\mathbf{k}_1+\mathbf{k}_2}^{\nu_3}}$$

Here, $E_{h\mathbf{k}}^{\nu_1}=(-1)^{\nu_1+1}E_{h\mathbf{k}}$, ν_1=1, 2. Then the self energy of charge carrier could be write in detail.

The self-consistent equation about charge carrier:

1、 $\phi_1 = < h^+_{i\sigma} h_{i+\hat{\eta}\sigma} > = \frac{1}{2N}\sum_k \gamma_k Z_F [1 - \frac{\bar{\xi}_k}{E_k}\tanh(\frac{1}{2}\beta E_k)]$

2、 $\phi_2 = < h^+_{i\sigma} h_{i+\hat{\tau}\sigma} > = \frac{1}{2N}\sum_k \gamma_k' Z_F [1 - \frac{\bar{\xi}_k}{E_k}\tanh(\frac{1}{2}\beta E_k)]$

3、 $\delta = < h^+_{i\sigma} h_{i\sigma} > = \frac{1}{2N}\sum_k Z_F [1 - \frac{\bar{\xi}_k}{E_k}\tanh(\frac{1}{2}\beta E_k)]$

4、 $\chi_1 = < S^+_i S^-_{i+\hat{\eta}} > = \frac{1}{N}\sum_k \gamma_k \frac{B_k}{2\omega_k}\coth(\frac{1}{2}\beta\omega_k)$

5、 $\chi_2 = < S^+_i S^-_{i+\hat{\tau}} > = \frac{1}{N}\sum_k \gamma_k' \frac{B_k}{2\omega_k}\coth(\frac{1}{2}\beta\omega_k)$

6、 $C_1 = \frac{1}{Z^2}\sum_{\hat{\eta}\hat{\eta}'} < S^+_{i+\hat{\eta}} S^-_{i+\hat{\eta}'} > = \frac{1}{N}\sum_k \gamma_k^2 \frac{B_k}{2\omega_k}\coth(\frac{1}{2}\beta\omega_k)$

7、 $C_2 = \frac{1}{Z^2}\sum_{\hat{\tau}\hat{\tau}'} < S^+_{i+\hat{\tau}} S^-_{i+\hat{\tau}'} > = \frac{1}{N}\sum_k \gamma_k'^2 \frac{B_k}{2\omega_k}\coth(\frac{1}{2}\beta\omega_k)$

8、 $C_3 = \frac{1}{Z^2}\sum_{\hat{\eta}\hat{\tau}} < S^+_{i+\hat{\eta}} S^-_{i+\hat{\tau}} > = \frac{1}{N}\sum_k \gamma_k \gamma_k' \frac{B_k}{2\omega_k}\coth(\frac{1}{2}\beta\omega_k)$

9、 $\frac{1}{2} = < S^+_i S^-_i > = \frac{1}{N}\sum_k \frac{B_k}{2\omega_k}\coth(\frac{1}{2}\beta\omega_k)$

10、 $\chi_1^z = < S^z_i S^z_{i+\hat{\eta}} > = \frac{1}{N}\sum_k \gamma_k \frac{B_z(k)}{2\omega_z(k)}\coth[\frac{1}{2}\beta\omega_z(k)]$

11、 $\chi_2^z = < S^z_i S^z_{i+\hat{\tau}} > = \frac{1}{N}\sum_k \gamma_k' \frac{B_z(k)}{2\omega_z(k)}\coth[\frac{1}{2}\beta\omega_z(k)]$

12、 $C_1^z = \frac{1}{Z^2}\sum_{\hat{\eta}\hat{\eta}'} < S^z_{i+\hat{\eta}} S^z_{i+\hat{\eta}'} > = \frac{1}{N}\sum_k \gamma_k^2 \frac{B_z(k)}{2\omega_z(k)}\coth[\frac{1}{2}\beta\omega_z(k)]$

13、 $C_3^z = \frac{1}{Z^2}\sum_{\hat{\eta}\hat{\tau}} < S^z_{i+\hat{\eta}} S^z_{i+\hat{\tau}} > = \frac{1}{N}\sum_k \gamma_k \gamma_k' \frac{B_z(k)}{2\omega_z(k)}\coth[\frac{1}{2}\beta\omega_z(k)]$

14、
$$1=\frac{1}{N^3}\sum_{kpq}(Zt\gamma_{p+k}+Zt'\gamma_{p+k}')^2\gamma_k^{(a)}\gamma_{k+p-q}^{(a)}Z_F{}^2\frac{1}{E_k}\cdot\frac{B_pB_q}{\omega_p\omega_q}\times$$
$$\{\frac{[1-2n_F(E_k)]\cdot[n_B(\omega_q)-n_B(\omega_p)]\cdot(\omega_p-\omega_q)-\{n_B(\omega_q)[1+n_B(\omega_p)]+n_B(\omega_p)[1+n_B(\omega_q)]\}\cdot E_k}{(\omega_q-\omega_p)^2-E_k{}^2}$$
$$+\frac{[1-2n_F(E_k)]\cdot[1+n_B(\omega_q)+n_B(\omega_p)]\cdot(\omega_q+\omega_p)-\{n_B(\omega_q)n_B(\omega_p)+[1+n_B(\omega_q)][1+n_B(\omega_p)]\}\cdot E_k}{(\omega_q+\omega_p)^2-E_k{}^2}\}$$

15、
$$Z_{hF}{}^{-1}=1-\Sigma_{ho}^{(1)}(k)$$
$$=1+\frac{1}{N^2}\sum_{pq}(Zt\gamma_{k+p}-Zt'\gamma'_{k+p})^2\cdot Z_F\cdot\frac{B_pB_q}{4\omega_p\omega_q}\times$$
$$\{\frac{n_B(\omega_p)[1+n_B(\omega_q)]+n_F(E_{k+p-q})[n_B(\omega_q)-n_B(\omega_p)]}{(E_{k+p-q}-\omega_p+\omega_q)^2}+\frac{[1+n_B(\omega_p)][1+n_B(\omega_q)]-n_F(E_{k+p-q})[1+n_B(\omega_p)+n_B(\omega_q)]}{(E_{k+p-q}+\omega_p+\omega_q)^2}$$
$$+\frac{n_B(\omega_p)n_B(\omega_q)+n_F(E_{k+p-q})[n_B(\omega_p)+1+n_B(\omega_q)]}{(E_{k+p-q}-\omega_p-\omega_q)^2}+\frac{n_B(\omega_q)[1+n_B(\omega_p)]+n_F(E_{k+p-q})[n_B(\omega_p)-n_B(\omega_q)]}{(E_{k+p-q}+\omega_p-\omega_q)^2}\}$$

Thus, all parameter of mean field could be got by solving the self-consistent equations, no any parameter added phenomenally.

Full charge-spin recombination:

Definition of Diagonal Green function and off-diagonal Green function in electronic representation:

$$G(i-j,t-t') = \langle\langle C_{i\sigma}(t); C^{\dagger}_{j\sigma}(t')\rangle\rangle$$

$$\Gamma(i-j,t-t') = \langle\langle C_{i\sigma}(t); C_{j\sigma}(t')\rangle\rangle$$

$$\Gamma^{\dagger}(i-j,t-t') = \langle\langle C^{\dagger}_{i\sigma}(t); C^{\dagger}_{j\sigma}(t')\rangle\rangle$$

Mean field Green function of electron: $G^{(0)}(\mathbf{k}, i\omega_m) = \dfrac{1}{i\omega_m - \varepsilon_{\mathbf{k}}}$

Here, the excitation spectrum： $\varepsilon_{\mathbf{k}} = -Zt\gamma_{\mathbf{k}} + Zt'\gamma'_{\mathbf{k}} + \mu,$

Dyson equations:

$$G(\mathbf{k}, i\omega_m) = G^{(0)}(\mathbf{k}, i\omega_m) + G^{(0)}(\mathbf{k}, i\omega_m)[\Sigma_{\mathrm{ph}}(\mathbf{k}, i\omega_m)G(\mathbf{k}, i\omega_m) - \Sigma_{\mathrm{pp}}(\mathbf{k}, -i\omega_m)\Gamma^{\dagger}(\mathbf{k}, -i\omega_m)],$$

$$\Gamma^{\dagger}(\mathbf{k}, i\omega_m) = G^{(0)}(-\mathbf{k}, -i\omega_m)[\Sigma_{\mathrm{ph}}(-\mathbf{k}, -i\omega_m)G(\mathbf{k}, i\omega_m) - \Sigma_{\mathrm{pp}}(-\mathbf{k}, -i\omega_m)\Gamma^{\dagger}(-\mathbf{k}, -i\omega_m)]$$

Here, the normal and abnormal self-energy:

$$\begin{aligned}
\Sigma_{\mathrm{ph}}(\mathbf{k}, i\omega_m) &= \frac{1}{N^2}\sum_{\mathbf{k}_1,\mathbf{k}_2}\Lambda^2_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}}\frac{1}{\beta^2}\sum_{i\omega_{m1},i\omega_{m2}} G(\mathbf{k}+\mathbf{k}_1, i\omega_m + i\omega_{m1})D^{(0)}(\mathbf{k}_1, i\omega_{m1}) \\
&\times D^{(0)}(\mathbf{k}_1+\mathbf{k}_2, i\omega_{m1}+i\omega_{m2}), \\
\Sigma_{\mathrm{pp}}(\mathbf{k}, i\omega_m) &= \frac{1}{N^2}\sum_{\mathbf{k}_1,\mathbf{k}_2}\Lambda^2_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}}\frac{1}{\beta^2}\sum_{i\omega_{m1},i\omega_{m2}} \Gamma^{\dagger}(-\mathbf{k}-\mathbf{k}_1, -i\omega_m - i\omega_{m1})D^{(0)}(\mathbf{k}_1, i\omega_{m1}) \\
&\times D^{(0)}(\mathbf{k}_1+\mathbf{k}_2, i\omega_{m1}+i\omega_{m2}),
\end{aligned}$$

Then, self-energy could be written as

$$\Sigma_{\rm ph}(\mathbf{k}, i\omega_m) = (-1)^{\nu_2+\nu_3} \frac{Z_F}{2N^2} \sum_{\substack{\mathbf{k}_1\mathbf{k}_2 \\ \nu_1\nu_2\nu_3}} \Lambda_{\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2}\left(1 + \frac{\bar{\varepsilon}_{\mathbf{k}+\mathbf{k}_2}}{E^{\nu_1}_{\mathbf{k}+\mathbf{k}_2}}\right)\frac{b_{\mathbf{k}_1} b_{\mathbf{k}_1+\mathbf{k}_2}}{4\omega_{\mathbf{k}_1}\omega_{\mathbf{k}_1+\mathbf{k}_2}} \times \frac{[1+n_B(\omega^{\nu_2}_{\mathbf{k}_1})]n_B(\omega^{\nu_3}_{\mathbf{k}_1+\mathbf{k}_2}) + n_F(E^{\nu_1}_{\mathbf{k}+\mathbf{k}_1})[n_B(\omega^{\nu_2}_{\mathbf{k}_1}) - n_B(\omega^{\nu_3}_{\mathbf{k}_1+\mathbf{k}_2})]}{i\omega_m - E^{\nu_1}_{\mathbf{k}+\mathbf{k}_1} - \omega^{\nu_2}_{\mathbf{k}_1} + \omega^{\nu_3}_{\mathbf{k}_1+\mathbf{k}_2}}$$

$$\Sigma_{\rm pp}(\mathbf{k}, i\omega_m) = (-1)^{\nu_2+\nu_3+1} \frac{1}{N^2} \sum_{\substack{\mathbf{k}_1\mathbf{k}_2 \\ \nu_1\nu_2\nu_3}} \Lambda_{\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2}\frac{\bar{\Delta}_Z(\mathbf{k}+\mathbf{k}_2)}{2E^{\nu_1}_{\mathbf{k}+\mathbf{k}_2}}\frac{b_{\mathbf{k}_1} b_{\mathbf{k}_1+\mathbf{k}_2}}{4\omega_{\mathbf{k}_1}\omega_{\mathbf{k}_1+\mathbf{k}_2}} \times \frac{[1+n_B(\omega^{\nu_2}_{\mathbf{k}_1})]n_B(\omega^{\nu_3}_{\mathbf{k}_1+\mathbf{k}_2}) + n_F(E^{\nu_1}_{\mathbf{k}+\mathbf{k}_1})[n_B(\omega^{\nu_2}_{\mathbf{k}_1}) - n_B(\omega^{\nu_3}_{\mathbf{k}_1+\mathbf{k}_2})]}{i\omega_m - E^{\nu_1}_{\mathbf{k}+\mathbf{k}_1} - \omega^{\nu_2}_{\mathbf{k}_1} + \omega^{\nu_3}_{\mathbf{k}_1+\mathbf{k}_2}}$$

Here, $E^{\nu_1}_{h\mathbf{k}} = (-1)^{\nu_1+1} E_{h\mathbf{k}}$, ν_1, ν_2, ν_3=1, 2 , then normal and anomalous self-energy is got in detail.

Full Green function in electronic representation:

In long wave limit approximation:

$$G(\mathbf{k}, i\omega_m) = Z_F\left(\frac{U_\mathbf{k}^2}{i\omega_m - E_\mathbf{k}} + \frac{V_\mathbf{k}^2}{i\omega_m + E_\mathbf{k}}\right),$$

$$\Gamma^\dagger(\mathbf{k}, i\omega_m) = -Z_F\frac{\bar{\Delta}_Z(\mathbf{k})}{2E_\mathbf{k}}\left(\frac{1}{i\omega_m - E_\mathbf{k}} - \frac{1}{i\omega_m + E_\mathbf{k}}\right).$$

其中:

$$U_\mathbf{k}^2 = \frac{1}{2}\left(1 + \frac{\bar{\varepsilon}_\mathbf{k}}{E_\mathbf{k}}\right)$$

$$V_\mathbf{k}^2 = \frac{1}{2}\left(1 - \frac{\bar{\varepsilon}_\mathbf{k}}{E_\mathbf{k}}\right)$$

Here, the renormalized excitation spectrum of electron is $\bar{\varepsilon}_{\boldsymbol{k}} = Z_F\varepsilon_{\boldsymbol{k}}$, abnormal excitation spectrum of electron is $E_{\boldsymbol{k}} = \sqrt{\bar{\varepsilon}_{\boldsymbol{k}}^2 + \bar{\Delta}_z^2(\boldsymbol{k})}$, and $\bar{\Delta}_z(\boldsymbol{k})$is the renormalized gap function, then $U_k^2 + V_k^2 = 1$.

Self-consistent equation in electronic representation:

1.
$$Z_F^{-1} = 1 + (-1)^{\nu_2+\nu_3}\frac{Z_F}{2N^2}\sum_{\substack{\mathbf{k}_1\mathbf{k}_2\\ \nu_1\nu_2\nu_3}}\Lambda_{\mathbf{k}_A+\mathbf{k}_1+\mathbf{k}_2}\left(1+\frac{\bar{\varepsilon}_{\mathbf{k}_A+\mathbf{k}_2}}{E^{\nu_1}_{\mathbf{k}_A+\mathbf{k}_2}}\right)\frac{b_{\mathbf{k}_1}b_{\mathbf{k}_1+\mathbf{k}_2}}{4\omega_{\mathbf{k}_1}\omega_{\mathbf{k}_1+\mathbf{k}_2}}$$
$$\times \frac{[1+n_B(\omega^{\nu_2}_{\mathbf{k}_1})]n_B(\omega^{\nu_3}_{\mathbf{k}_1+\mathbf{k}_2}) + n_F(E^{\nu_1}_{\mathbf{k}_A+\mathbf{k}_1})[n_B(\omega^{\nu_2}_{\mathbf{k}_1}) - n_B(\omega^{\nu_3}_{\mathbf{k}_1+\mathbf{k}_2})]}{(E^{\nu_1}_{\mathbf{k}_A+\mathbf{k}_1} + \omega^{\nu_2}_{\mathbf{k}_1} - \omega^{\nu_3}_{\mathbf{k}_1+\mathbf{k}_2})^2}$$

2.
$$1 = (-1)^{\nu_2+\nu_3}\frac{4}{N^3}\sum_{\substack{\mathbf{k}\mathbf{k}_1\mathbf{k}_2\\ \nu_1\nu_2\nu_3}} Z_F\Lambda_{\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2}\frac{\gamma^{(d)}_{\mathbf{k}}\gamma^{(d)}_{\mathbf{k}+\mathbf{k}_2}}{2E^{\nu_1}_{\mathbf{k}+\mathbf{k}_2}}\frac{b_{\mathbf{k}_1}b_{\mathbf{k}_1+\mathbf{k}_2}}{4\omega_{\mathbf{k}_1}\omega_{\mathbf{k}_1+\mathbf{k}_2}}$$
$$\times \frac{[1+n_B(\omega^{\nu_2}_{\mathbf{k}_1})]n_B(\omega^{\nu_3}_{\mathbf{k}_1+\mathbf{k}_2}) + n_F(E^{\nu_1}_{\mathbf{k}+\mathbf{k}_1})[n_B(\omega^{\nu_2}_{\mathbf{k}_1}) - n_B(\omega^{\nu_3}_{\mathbf{k}_1+\mathbf{k}_2})]}{E^{\nu_1}_{\mathbf{k}+\mathbf{k}_1} + \omega^{\nu_2}_{\mathbf{k}_1} - \omega^{\nu_3}_{\mathbf{k}_1+\mathbf{k}_2}}$$

3.
$$1-\delta = \frac{1}{N}\sum_{\mathbf{k}} Z_F\left[1 - \frac{\bar{\varepsilon}_{\mathbf{k}}}{E_{\mathbf{k}}}\tanh(\frac{1}{2}\beta E_{\mathbf{k}})\right]$$

Here,μ is chemical potential, Z_F is renormalized factor and Δ is the superconducting gap in the electron representation, not any parameter is involved phenomenally.

The full Green function：

$$G(\mathbf{k}, i\omega_m) = \frac{1}{i\omega_m - \varepsilon_{\mathbf{k}} - \Sigma_{\text{tot}}(\mathbf{k}, i\omega_m)}$$

$$\mathfrak{I}^{\dagger}(\mathbf{k}, i\omega_m) = \frac{L(\mathbf{k}, i\omega_m)}{i\omega_m - \varepsilon_{\mathbf{k}} - \Sigma_{\text{tot}}(\mathbf{k}, i\omega_m)}$$

here， the self energy $\sum_{\text{tot}}(\boldsymbol{k}, i\omega_m)$:

$$\Sigma_{\text{tot}}(\mathbf{k}, i\omega_m) = \Sigma_{\text{ph}}(\mathbf{k}, i\omega_m) + \frac{|\Sigma_{\text{pp}}(\mathbf{k}, i\omega_m)|^2}{i\omega_m + \varepsilon_{\mathbf{k}} + \Sigma_{\text{ph}}(\mathbf{k}, -i\omega_m)}$$

$$L(\mathbf{k}, i\omega_m) = -\frac{\Sigma_{\text{pp}}(\mathbf{k}, i\omega_m)}{i\omega_m + \varepsilon_{\mathbf{k}} + \Sigma_{\text{ph}}(\mathbf{k}, -i\omega_m)}$$

Ⅲ The kink of dispersion

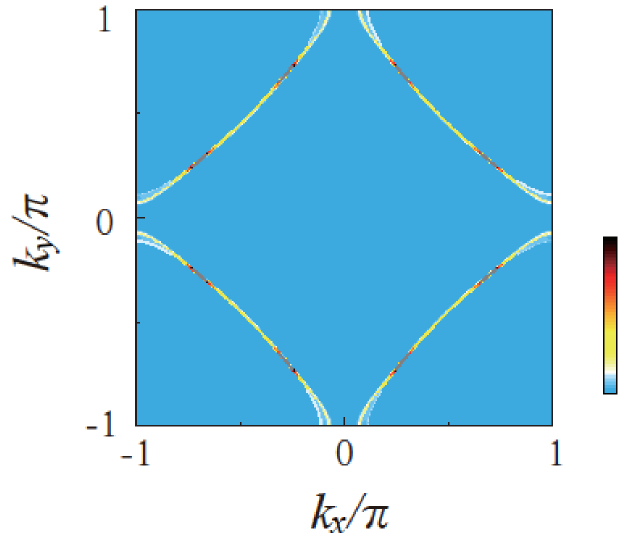
Fermi surface in superconducting state

Spectrum function of electron:

$$A(\mathbf{k},\omega) = \frac{-2\,\mathrm{Im}\,\Sigma_{\mathrm{tot}}(\mathbf{k},\omega)}{[\omega - \varepsilon_{\mathbf{k}} - \mathrm{Re}\,\Sigma_{\mathrm{tot}}(\mathbf{k},\omega)]^2 + [\mathrm{Im}\,\Sigma_{\mathrm{tot}}(\mathbf{k},\omega)]^2}$$

The location of the Fermi surface is determined by

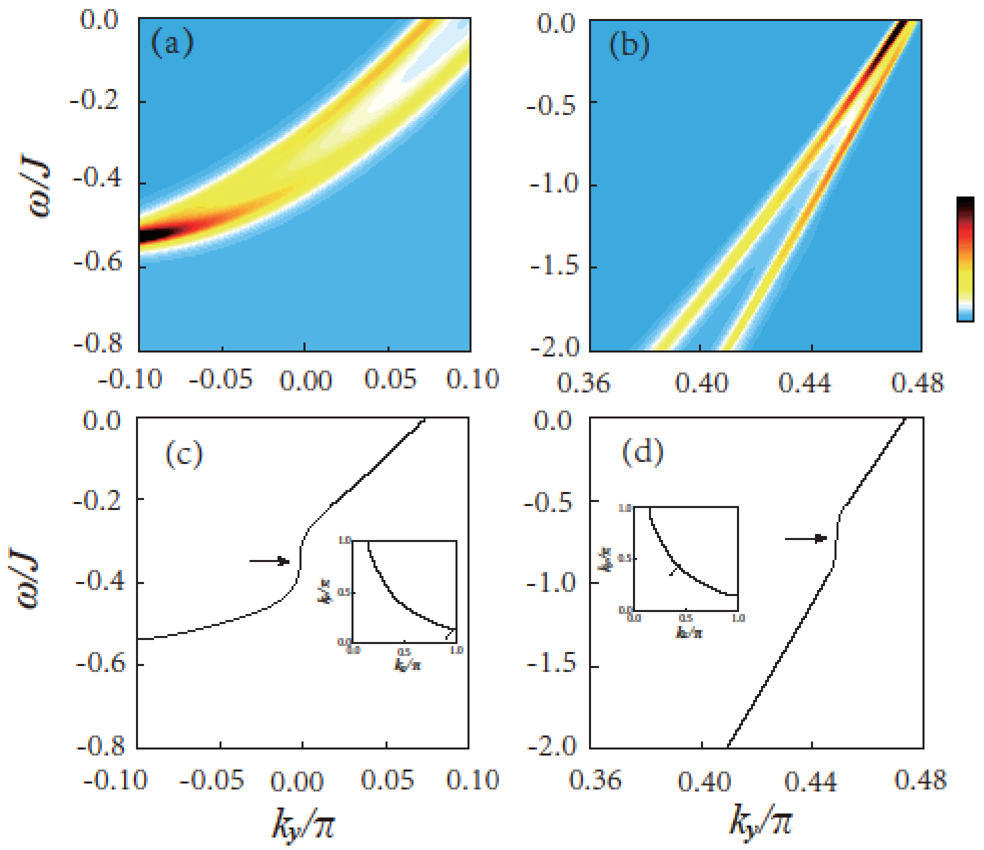
$$\varepsilon_{\mathbf{k}} + \mathrm{Re}\,\Sigma_{\mathrm{tot}}(\mathbf{k},0) = 0$$



The location of Fermi surface of cuprate superconductor 。 And doping δ= 0.15， T = 0.002J， t/J = 2.5 and t′/t = 0.1。

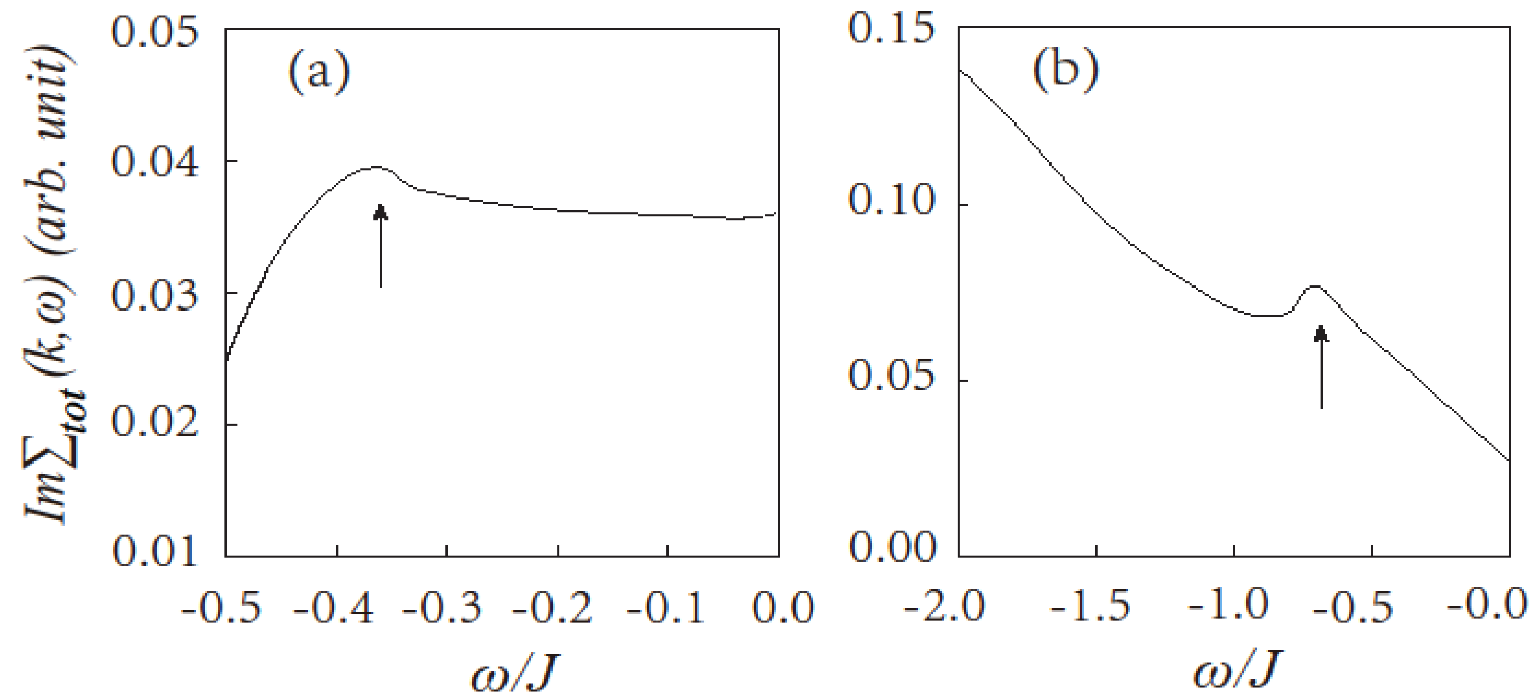
Kink structure of dispersion

dispersion: $\omega - \varepsilon_{\mathbf{k}} - \Sigma_{\text{tot}}(\mathbf{k}, \omega) = 0$



fig（a）and fig（b）is the strength of quasiparticle spectrum in the direction of $\frac{\pi}{4}$ at the antinodal and nodal point respectively; fig（c）and fig（d）are the dispersion at the antinodal and nodal point respectively, δ= 0.15， T= 0.002J， and t/J = 2.5, t′/t = 0.1。The kink energy of antinodal point ω_{kink}^{AN}≈ 0.34J = 34meV and the kink energy of nodal point ω_{kink}^{ND}≈ 0.73J = 73meV。

Imaginary and real part of self energy in the dispersion:



fig（a）is the imaginary part of total self energy of electronic dispersion，fig（b）is the real part of total self energy of electronic dispersion ，δ= 0.15， T = 0.002J，and t/J = 2.5，t'/t = 0.1，the location of arrow is the kink energy of electronic dispersion。

The peak-dip-hump structure of energy distribution curve(EDC)

ARPES quasiparticle spectrum： $I(\mathbf{k},\omega) = |M_{IF}(\mathbf{k},\omega)| n_F(\omega) A(\mathbf{k},\omega)$

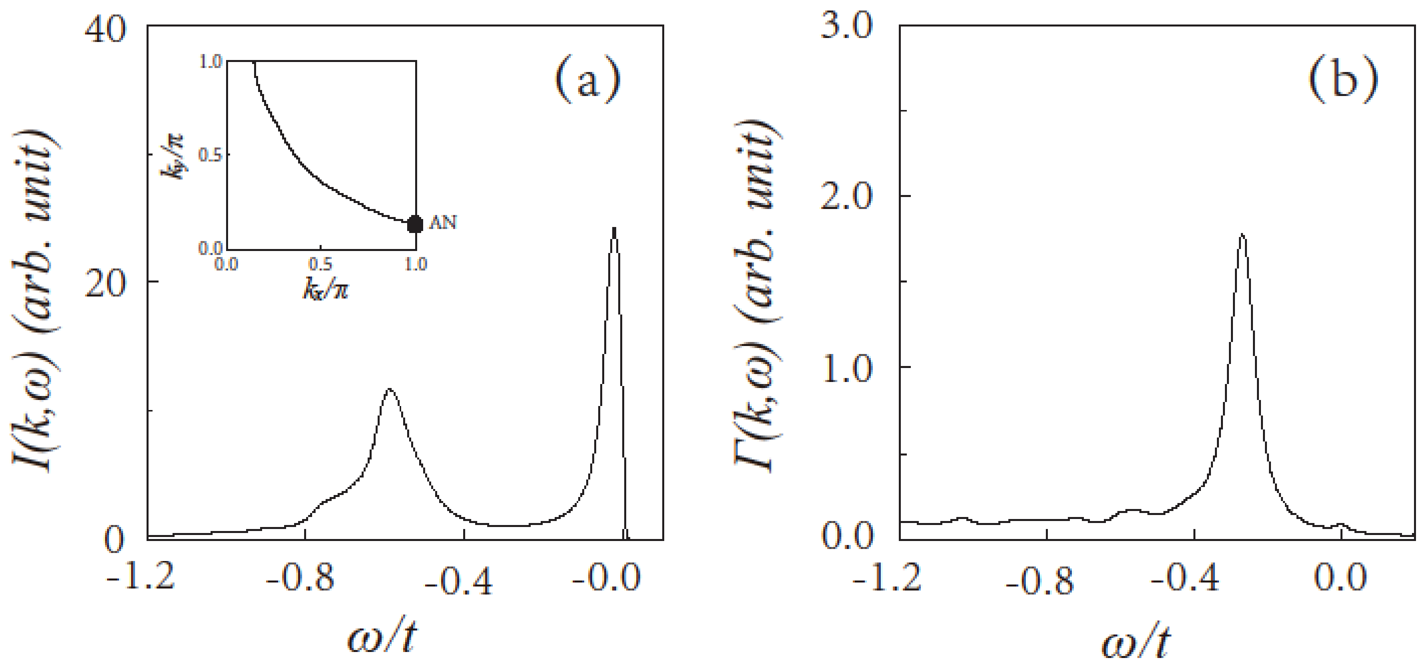


fig (a) is PDH of the energy distribution at the antinodal point， fig (b) is the imaginary part of self energy corresponding to (a)。 And, $t/J = 2.5$， $t'/t=0.3$， $T = 0.002J$, $\delta = 0.15$。

Conclusion:

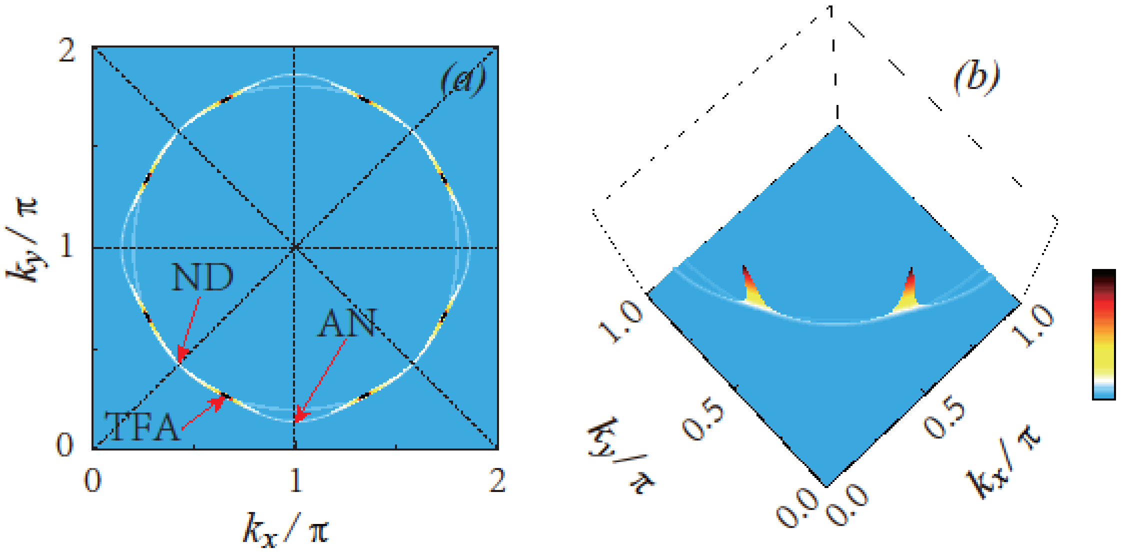
The same reason lead to the kink structure of dispersion and the peak-dip-hump（PDH） structure， which is the unusual distribution of the self energy。

IV T-linear resistivity in the strange metal phase

Fermi surface in the normal state

Spectrum function:

$$A(\mathbf{k},\omega) = -2\mathrm{Im}G(\mathbf{k},\omega) = -\frac{2\mathrm{Im}\Sigma_{\mathrm{ph}}(\mathbf{k},\omega)}{[\omega-\varepsilon_{\mathbf{k}}-\mathrm{Re}\Sigma_{\mathrm{ph}}(\mathbf{k},\omega)]^2+[\mathrm{Im}\Sigma_{\mathrm{ph}}(\mathbf{k},\omega)]^2}$$

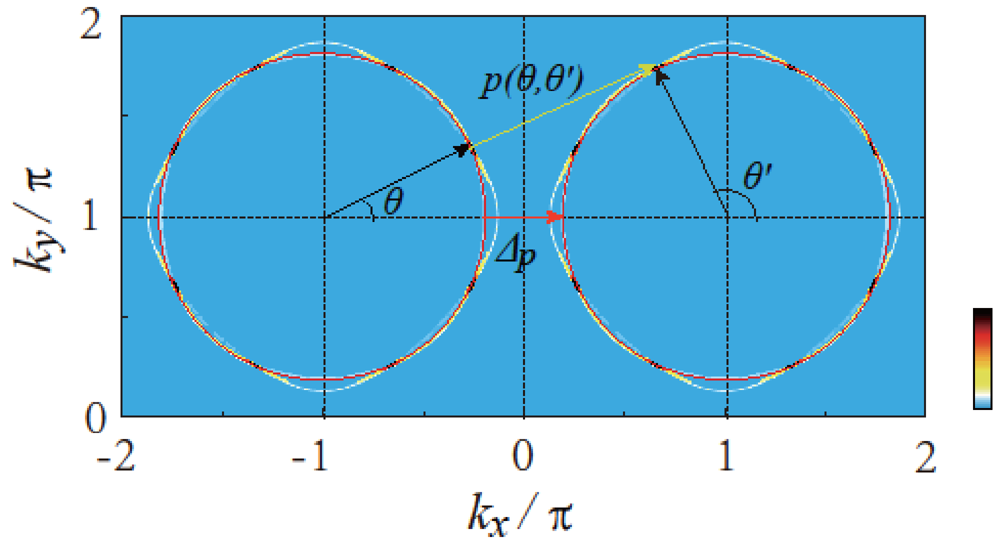


The location of Fermi surface is decided by the equation:

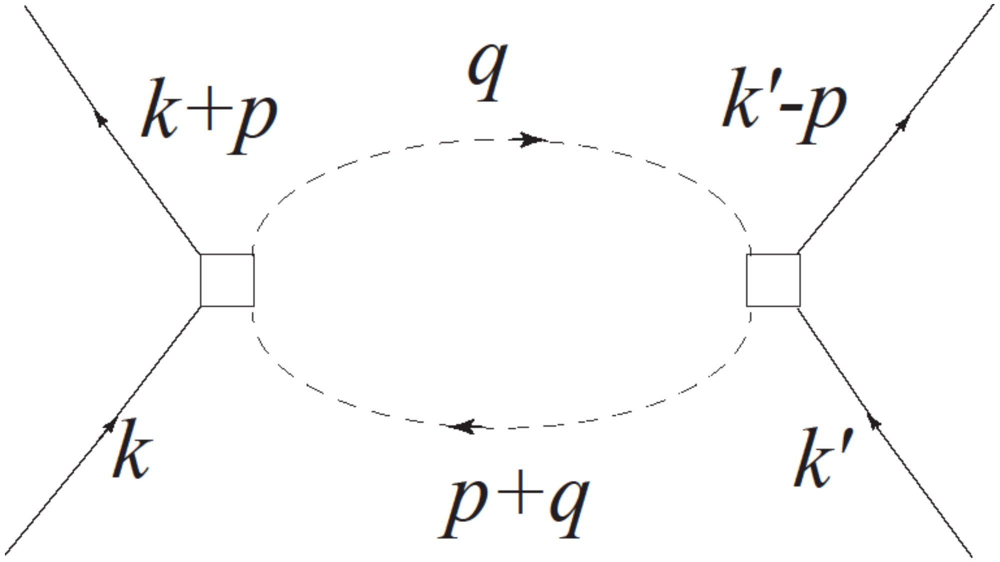
$$\varepsilon_{\mathbf{k}} + \mathrm{Re}\Sigma_{\mathrm{ph}}(\mathbf{k},0) = 0$$

fig （a） is the weight of electronic quasiparticle spectrum。 fig （b） is the 3D distribution corresponding to (a) 。Then, δ= 0.18, T = 0.002J， and t/J = 2.5, t’/t = 0.3。

Umklapp scattering process



Electron scattering occurs between different Brillouin zones , Minimum scattering wave vector is Δ_p



Umklapp scattering process between electrons, The solid line represents the electron propagator *G*, and the dashed line depicts the spin propagator *D*(0), while □ describes the bare vertex function .

According to the skeletal diagram of the umklapp scattering process, the electrons scattering by exchange an effective spin propagator, and the propagator could be written as

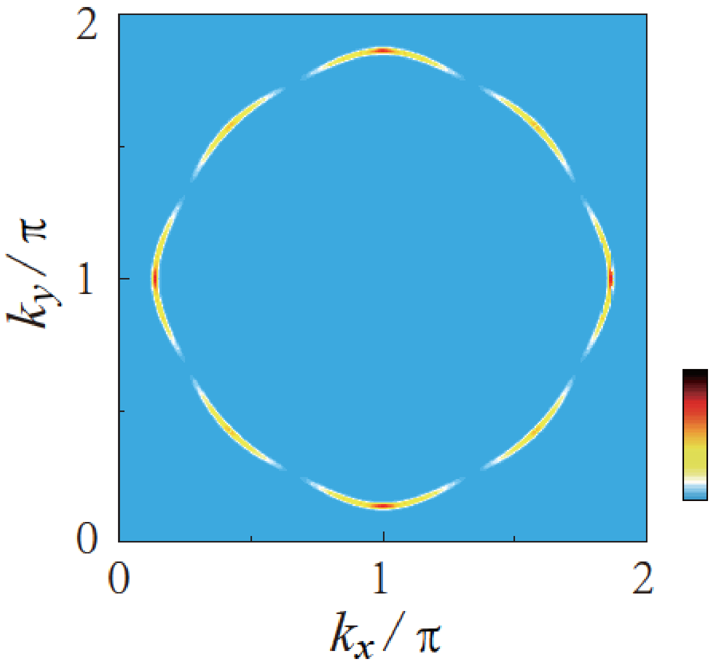
$$P(\mathbf{k},\mathbf{p},\mathbf{k}',ip_m) = \frac{1}{N}\sum_{\mathbf{q}}\Lambda_{\mathbf{p}+\mathbf{q}+\mathbf{k}}\Lambda_{\mathbf{q}+\mathbf{k}'}\frac{1}{\beta}\sum_{iq_m}D^0(\mathbf{q},iq_m)D^0(\mathbf{p}+\mathbf{q},ip_m+iq_m)$$

Here, the scattering electron momentum is **k** and **k'**, where they are scattered to the state with momentum **k+p** and **k'-p** respectively. Then sum of the internal frequencies, the propagator could be expressed as follows:

$$P(\mathbf{k},\mathbf{p},\mathbf{k}',ip_m) = \frac{2}{N}\sum_{\mathbf{q}}\Lambda_{\mathbf{p}+\mathbf{q}+\mathbf{k}}\Lambda_{\mathbf{q}+\mathbf{k}'}\frac{B_{\mathbf{q}}B_{\mathbf{p}+\mathbf{q}}}{2\omega_{\mathbf{q}}\omega_{\mathbf{p}+\mathbf{q}}} \times\left\{\frac{[n_B(\omega_{\mathbf{q}})-n_B(\omega_{\mathbf{p}+\mathbf{q}})](\omega_{\mathbf{q}}-\omega_{\mathbf{p}+\mathbf{q}})}{ip_m^2-(\omega_{\mathbf{q}}-\omega_{\mathbf{p}+\mathbf{q}})^2}-\frac{[n_B(\omega_{\mathbf{q}})+n_B(\omega_{\mathbf{p}+\mathbf{q}})+1](\omega_{\mathbf{q}}+\omega_{\mathbf{p}+\mathbf{q}})}{ip_m^2-(\omega_{\mathbf{q}}+\omega_{\mathbf{p}+\mathbf{q}})^2}\right\}$$

Effective spin excitation propagator

$$P(\mathbf{k}, \mathbf{p}-\mathbf{k}, \mathbf{k}', \omega) = \frac{1}{N}\sum_{\mathbf{q}} \Lambda_{\mathbf{p}+\mathbf{q}}\Lambda_{\mathbf{q}+\mathbf{k}'}\Pi(\mathbf{p}-\mathbf{k}, \mathbf{q}, \omega)$$
$$= \int_{-\infty}^{\infty} \frac{d\omega'}{\pi} \frac{\mathrm{Im}P(\mathbf{k}, \mathbf{p}-\mathbf{k}, \mathbf{k}', \omega')}{\omega' - \omega},$$



The imaginary part of the effective spin excitation propagator ImP(k,p−k,k′,ω), represent the spectrum strength of the propagator in the momentum space, this will help us learn more about contribution of different electron with different momentum 。

The distribution of Imaginary part of spin excitation propagator ImP(k,p−k,k′,ω) in (p_x ,p_y) plane, t/J = 2.5， t′/t = 0.3， and T = 0.002J δ= 0.18， where ω= 0.05J。

Boltzmann equation

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v_k} \cdot \nabla_{\mathbf{r}} f + \frac{\partial \mathbf{k}}{\partial t} \cdot \nabla_{\mathbf{k}} f - \left(\frac{df}{dt}\right)_{collisions} = 0$$

First and second term are zero, and $\frac{\partial \mathbf{k}}{\partial t} = -e\mathbf{E}.$ Then, $e\mathbf{E} \cdot \nabla_{\mathbf{k}} f(\mathbf{k}) = \left(\frac{df}{dt}\right)_{collisions}$

The distribution function of electron could be written as: $f(\mathbf{k}) = n_F(\bar{\varepsilon}_{\mathbf{k}}) - \frac{dn_F(\bar{\varepsilon}_{\mathbf{k}})}{d\bar{\varepsilon}_{\mathbf{k}}} \tilde{\Phi}(\mathbf{k})$

where $\tilde{\Phi}(\mathbf{k})$ has the interpretation of a local shift of the chemical potential, then Boltzmann equation is

$$e\mathbf{v_k} \cdot \mathbf{E} \frac{\partial n_F(\bar{\varepsilon}_{\mathbf{k}})}{\partial \bar{\varepsilon}_{\mathbf{k}}} = -\left(\frac{df}{dt}\right)_{collisions} = I_{ee}.$$

The calculation process about the electron umklapp scattering term I_{ee}

$$I_{ee} = \int \frac{d^2\mathbf{p}d^2\mathbf{k}'}{(2\pi)^2}\left|P(\mathbf{k},\mathbf{p},\mathbf{k}',\bar{\varepsilon}_{\mathbf{k}}-\bar{\varepsilon}_{\mathbf{k+p+G}})\right|^2\Big\{f(\mathbf{k})f(\mathbf{k}')[1-f(\mathbf{k}+\mathbf{p}+\mathbf{G})][1-f(\mathbf{k}'-\mathbf{p})]$$
$$-f(\mathbf{k}+\mathbf{p}+\mathbf{G})f(\mathbf{k}'-\mathbf{p})[1-f(\mathbf{k})][1-f(\mathbf{k}')]\Big\}\delta(\bar{\varepsilon}_{\mathbf{k}}+\bar{\varepsilon}_{\mathbf{k}'}-\bar{\varepsilon}_{\mathbf{k+p+G}}-\bar{\varepsilon}_{\mathbf{k}'-\mathbf{p}}).$$

where , $|P(\mathbf{k},\mathbf{p}-\mathbf{k},\mathbf{k}',\bar{\varepsilon}_{\mathbf{k}}-\bar{\varepsilon}_{\mathbf{p+G}})|^2$ is scattering probability, δ function ensures conservation of energy.

In the absence of an external field, distribution function is Fermi distribution: $f(\mathbf{k}) = n_F(\bar{\varepsilon}_{\mathbf{k}})$

And the scattering collision term is zero, $I_{ee} = 0$

The relationship is always valid：

$$n_F(\bar{\varepsilon}_{\mathbf{k}})n_F(\bar{\varepsilon}_{\mathbf{k}'})[1-n_F(\bar{\varepsilon}_{\mathbf{k+p+G}})][1-n_F(\bar{\varepsilon}_{\mathbf{k}'-\mathbf{p}})] = n_F(\bar{\varepsilon}_{\mathbf{k+p+G}})n_F(\bar{\varepsilon}_{\mathbf{k}'-\mathbf{p}})[1-n_F(\bar{\varepsilon}_{\mathbf{k}})][1-n_F(\bar{\varepsilon}_{\mathbf{k}'})]$$

Then, the scattering collision could be written as：

$$I_{ee} = \frac{1}{N^2}\sum_{\mathbf{k}',\mathbf{p}}(2/T)|P(\mathbf{k},\mathbf{p},\mathbf{k}',\bar{\varepsilon}_{\mathbf{k}}-\bar{\varepsilon}_{\mathbf{k+p+G}})|^2[\tilde{\Phi}(\mathbf{k})+\tilde{\Phi}(\mathbf{k}')-\tilde{\Phi}(\mathbf{k}+\mathbf{p}+\mathbf{G})-\tilde{\Phi}(\mathbf{k}'-\mathbf{p})]$$
$$\times n_F(\bar{\varepsilon}_{\mathbf{k}})[1-n_F(\bar{\varepsilon}_{\mathbf{k+p+G}})]n_F(\bar{\varepsilon}_{\mathbf{k}'})[1-n_F(\bar{\varepsilon}_{\mathbf{k}'-\mathbf{p}})]\delta(\bar{\varepsilon}_{\mathbf{k}}+\bar{\varepsilon}_{\mathbf{k}'}-\bar{\varepsilon}_{\mathbf{k+p+G}}-\bar{\varepsilon}_{\mathbf{k}'-\mathbf{p}}),$$

The sum over momentum is restricted to the Fermi surface and the two-dimensional sum over momentum is transformed into an integral over the Fermi angle, thus obtaining the following relatively simplified Boltzmann equation:

$$e\mathbf{v_F}(\theta)\cdot\mathbf{E} = -\int \frac{d\theta'}{2\pi} 2\nu(\theta')F(\theta,\theta')[\Phi(\theta)-\Phi(\theta')]$$

where, $\nu(\theta') = \frac{k_F k_F(\theta')}{4\pi^2 v_F^2 v_F(\theta')}$ is the density of states factor 。 $F(\theta,\theta')$ is the scattering kernel function,

$$F(\theta,\theta') = \int \frac{\omega^2}{T|\mathbf{p}(\theta')-\mathbf{k}(\theta)|}|\bar{P}(\mathbf{k},\mathbf{p}-\mathbf{k},\omega)|^2 n_B(\omega)[1+n_B(\omega)]\frac{d\omega}{2\pi}$$

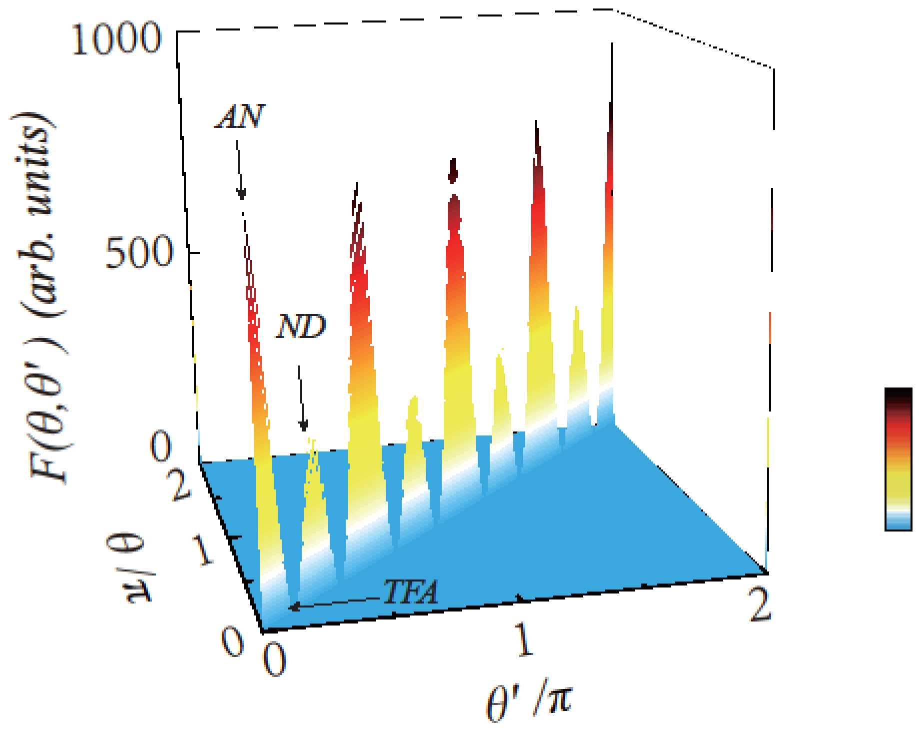
Transport scattering rate： $\gamma(\theta) = \int 2\nu(\theta')F(\theta,\theta')\frac{d\theta'}{2\pi}.$

Relaxing time approximation： $\Phi(\theta') = \Phi(\pi-\theta) = -\Phi(\theta),$

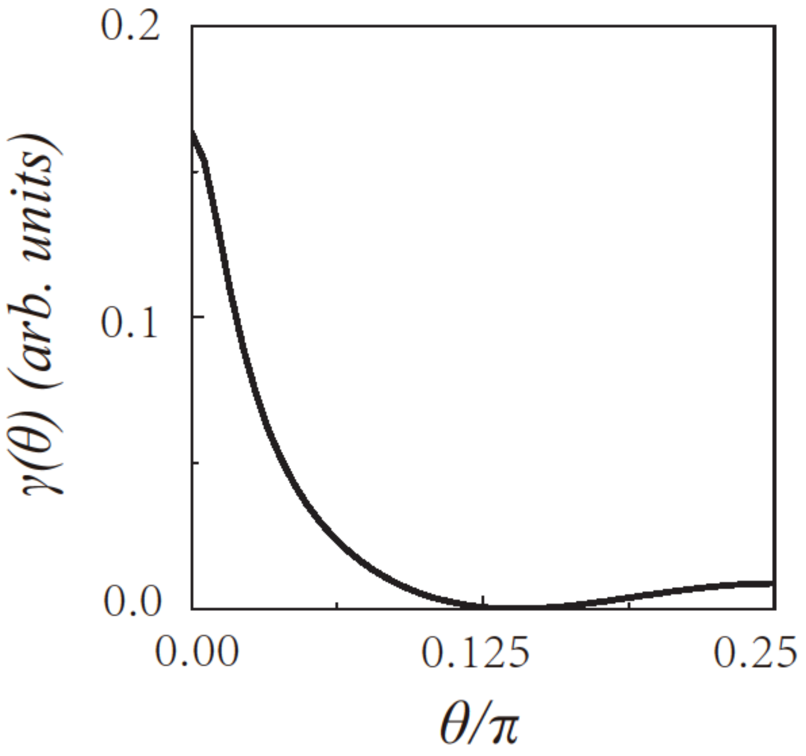
The solution Boltzmann equation in the relaxing time approximation:

$$\Phi(\theta) = -\frac{e\mathrm{v_F}\cos(\theta)E_{\hat{x}}}{2\gamma(\theta)}.$$

Scattering kernel function and transport scattering rate



Scattering kernel function as a function of Fermi angle, where, AN marked the maximum at the antinodal region , ND marked another peak at the nodal region, TFA is the tips of Fermi arc., with t/J = 2.5, t′/t = 0.3, and T = 0.05J, δ= 0.18。



Transport scattering rate as a function of scattering angle with t/J = 2.5, t′/t = 0.3, and T = 0.05J, δ= 0.18 .

Current function：

$$\mathbf{J} = en_0 \frac{1}{N} \sum_{\mathbf{k}} \mathbf{v}_{\mathbf{k}} \frac{dn_{\mathrm{F}}(\bar{\varepsilon}_{\mathbf{k}})}{d\bar{\varepsilon}_{\mathbf{k}}} \tilde{\Phi}(\mathbf{k})$$
$$= -en_0 \frac{\mathrm{k_F}}{\mathrm{v_F}} \int \frac{d\theta}{(2\pi)^2} \mathbf{v}_{\mathrm{F}}(\theta) \Phi(\theta).$$

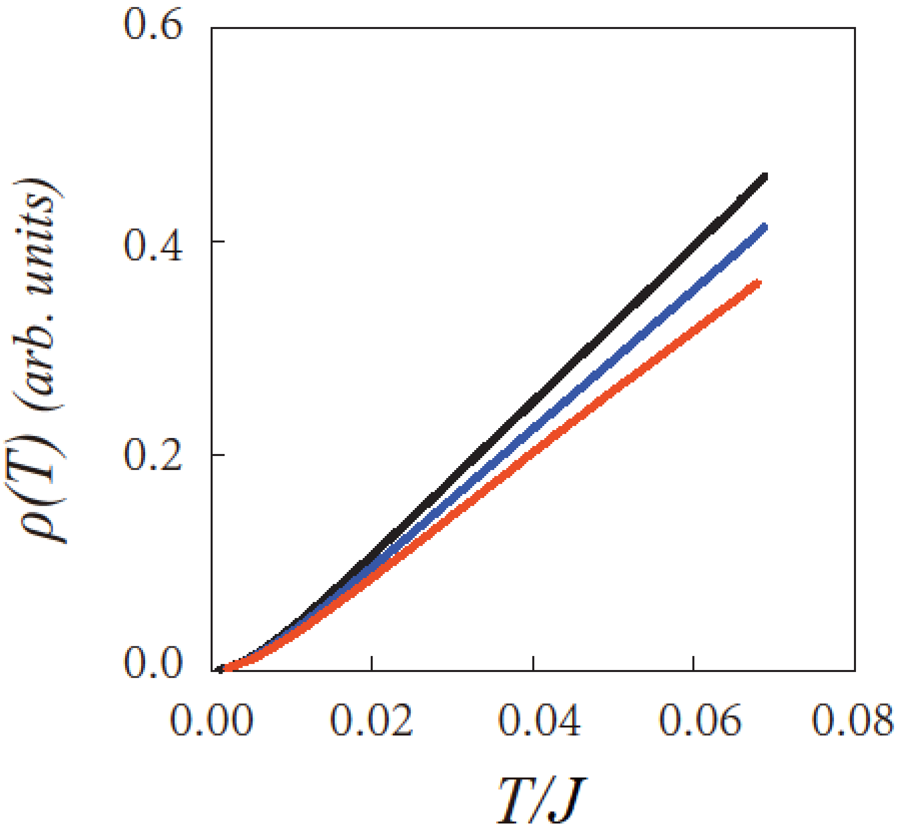
Substituting the solution of the Boltzmann equation into the current function, we obtain the expression for the DC conductivity.：

$$\sigma_{\mathrm{dc}}(T) = \frac{1}{2} e^2 n_0 \mathrm{k_F} \mathrm{v_F} \int \frac{d\theta}{(2\pi)^2} \cos^2(\theta) \frac{1}{\gamma(\theta)},$$

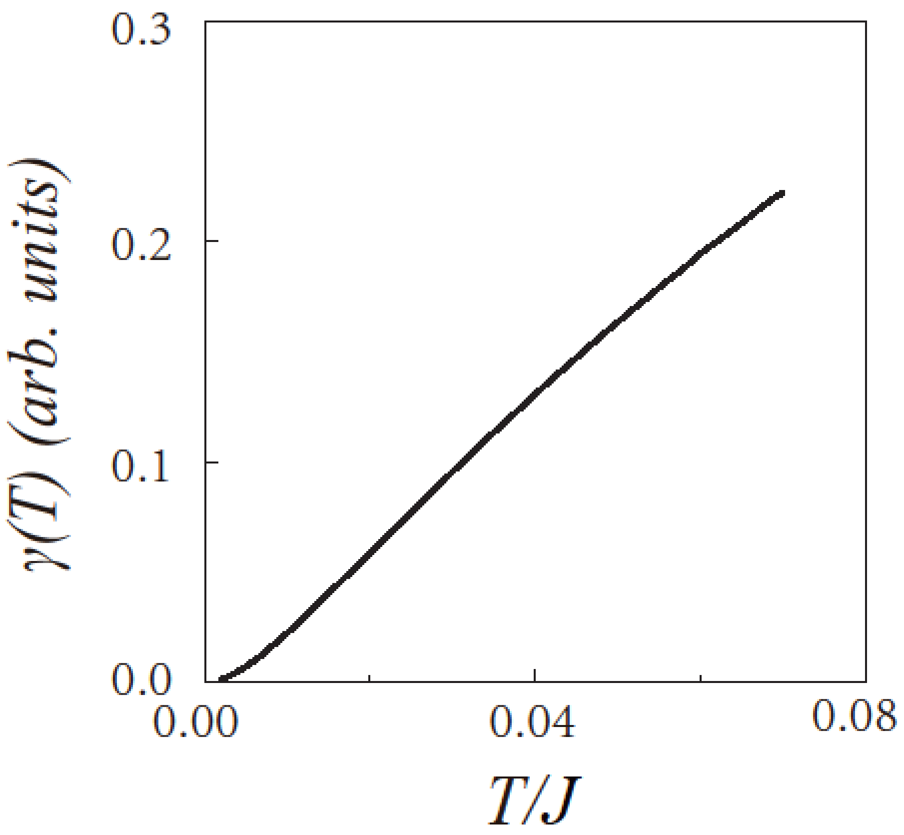
And resistivity：

$$\rho(T) = \frac{1}{\sigma_{\mathrm{dc}}(T)}.$$

T-linear resistivity and scattering rate



Resistivity as a function of temperature, t/J = 2.5, t′/t = 0.3, black line is δ=0.15, blue line is δ=0.18, red line is δ= 0.24。



The scattering rate at the antinodal region as a function of temperature , with t/J = 2.5, t′/t = 0.3, and δ= 0.18 。

Characteristic temperature : $T_{\text{scale}} = \bar{a}\Delta_p^2$

where: $\bar{a} = (1/N)\sum_{\mathbf{q}} a(\mathbf{q})$, $a(\mathbf{q}) = d^2\omega_{\mathbf{q}}/d^2\mathbf{q}$

By approximately calculating the energy integration of the scattering kernel function, we draw the following conclusions: When the temperature is higher than the characteristic temperature T_{scale}, the scattering kernel function varies linearly with temperature; when it is lower than the characteristic temperature T_{scale}, it depends on temperature quadratically.

conclusion：

The transport behavior of cuprate high-temperature superconductors in the overdoped strange metal phase is caused by the umklapp scattering between electrons, resulting from the exchange of an effective spin propagator. Below the characteristic temperature T_{scale}, the resistivity exhibits a quadratic dependence on temperature, while above the characteristic temperature T_{scale}, the resistivity shows a linear dependence on temperature.

V T-linear resistivity and superconductivity

Hamilton of electron doping cuprate：

$$H = -t\sum_{i\hat{\eta}\sigma} C^{\dagger}_{i\sigma}C_{i+\hat{\eta}\sigma} + t'\sum_{i\hat{\tau}\sigma} C^{\dagger}_{i\sigma}C_{i+\hat{\tau}\sigma} + \mu\sum_{i\sigma} C^{\dagger}_{i\sigma}C_{i\sigma} + J\sum_{i\hat{\eta}} \hat{\mathbf{S}}_i\cdot\hat{\mathbf{S}}_{i+\hat{\eta}},$$

Electron no zero occupation constrain： $\sum_{\sigma} C^{\dagger}_{i\sigma}C_{i\sigma} \geq 1.$

Particle-hole symmetry transform： $C_{i\sigma} \rightarrow f^{\dagger}_{i-\sigma}$

Hamilton in the hole representation：

$$H = t\sum_{i\hat{\eta}\sigma} f^{\dagger}_{i\sigma}f_{i+\hat{\eta}\sigma} - t'\sum_{i\hat{\tau}\sigma} f^{\dagger}_{i\sigma}f_{i+\hat{\tau}\sigma} - \mu_{\mathrm{f}}\sum_{i\sigma} f^{\dagger}_{i\sigma}f_{i\sigma} + J\sum_{i\hat{\eta}} \hat{\mathbf{S}}_i\cdot\hat{\mathbf{S}}_{i+\hat{\eta}}$$

Constrain in the hole representation： $\sum_{\sigma} f^{\dagger}_{i\sigma}f_{i\sigma} \leq 1$

The same treatment is applied to the Hamiltonian as hole doping cuprate

1. Separate the charge degree of freedom and the spin degrees of freedom through the fermion-spin theory；
2. The charge and spin Green functions are obtained under the mean field approximation, ；
3. The normal and anomalous self energy are obtained through Eliashberg strong coupling theory；
4. By applying Green function motion equation and using the mean-field self-consistent method, all self-consistent parameters can be obtained.

$$G_{\mathrm{f}}(\mathbf{k}, i\omega_m) = \frac{1}{i\omega_m - \varepsilon_{\mathrm{f}\mathbf{k}} - \Sigma^{(\mathrm{f})}_{\mathrm{tot}}(\mathbf{k}, i\omega_m)}$$

$$\Gamma^{\dagger}_{\mathrm{f}}(\mathbf{k}, i\omega_m) = \frac{L_{\mathrm{f}}(\mathbf{k}, i\omega_m)}{i\omega_m - \varepsilon_{\mathrm{f}\mathbf{k}} - \Sigma^{(\mathrm{f})}_{\mathrm{tot}}(\mathbf{k}, i\omega_m)}$$

and:

$$\Sigma^{(\mathrm{f})}_{\mathrm{tot}}(\mathbf{k}, i\omega_m) = \Sigma^{(\mathrm{f})}_{\mathrm{ph}}(\mathbf{k}, i\omega_m) + \frac{|\Sigma^{(\mathrm{f})}_{\mathrm{pp}}(\mathbf{k}, i\omega_m)|^2}{i\omega_m + \varepsilon_{\mathrm{f}\mathbf{k}} + \Sigma^{(\mathrm{f})}_{\mathrm{ph}}(\mathbf{k}, -i\omega_m)}$$

$$L_{\mathrm{f}}(\mathbf{k}, i\omega_m) = -\frac{\Sigma^{(\mathrm{f})}_{\mathrm{pp}}(\mathbf{k}, i\omega_m)}{i\omega_m + \varepsilon_{\mathrm{f}\mathbf{k}} + \Sigma^{(\mathrm{f})}_{\mathrm{ph}}(\mathbf{k}, -i\omega_m)}$$

Particle-hole symmetry:

$$G(l-l',t-t') = \langle\langle C_{l\sigma}(t); C^{\dagger}_{l'\sigma}(t')\rangle\rangle = \langle\langle f^{\dagger}_{l\sigma}(t); f_{l'\sigma}(t')\rangle\rangle = -G_{\mathrm{f}}(l'-l,t'-t),$$

$$\Im(l-l',t-t') = \langle\langle C_{l\downarrow}(t); C_{l'\uparrow}(t')\rangle\rangle = \langle\langle f^{\dagger}_{l\uparrow}(t); f^{\dagger}_{l'\downarrow}(t')\rangle\rangle = \Im^{\dagger}_{\mathrm{f}}(l-l',t-t')。$$

Full Green function in hole representation：

$$G(\mathbf{k},\omega) = \frac{1}{\omega - \varepsilon_{\mathbf{k}} - \Sigma_{\mathrm{tot}}(\mathbf{k},\omega)},$$

$$\Im^{\dagger}(\mathbf{k},\omega) = \frac{L(\mathbf{k},\omega)}{\omega - \varepsilon_{\mathbf{k}} - \Sigma_{\mathrm{tot}}(\mathbf{k},\omega)},$$

where

$$\Sigma_{\mathrm{tot}}(\mathbf{k},\omega) = \Sigma_{\mathrm{ph}}(\mathbf{k},\omega) + \frac{|\Sigma_{\mathrm{pp}}(\mathbf{k},\omega)|^2}{\omega + \varepsilon_{\mathbf{k}} + \Sigma_{\mathrm{ph}}(\mathbf{k},-\omega)},$$

$$L(\mathbf{k},\omega) = -\frac{\Sigma_{\mathrm{pp}}(\mathbf{k},\omega)}{\omega + \varepsilon_{\mathbf{k}} + \Sigma_{\mathrm{ph}}(\mathbf{k},-\omega)},$$

and

$$\Sigma_{\mathrm{ph}}(\mathbf{k},\omega) = -\Sigma^{(\mathrm{f})}_{\mathrm{ph}}(\mathbf{k},-\omega),$$

$$\Sigma_{\mathrm{pp}}(\mathbf{k},\omega) = \Sigma^{(\mathrm{f})}_{\mathrm{pp}}(\mathbf{k},\omega).$$

Excitation spectrum：

$$\varepsilon_{\mathbf{k}} = -\varepsilon_{\mathrm{f}\mathbf{k}} = 4t\gamma_{\mathbf{k}} - 4t'\gamma'_{\mathbf{k}} - \mu_{\mathrm{f}},$$

Fermi surface of electron doping cuprate：

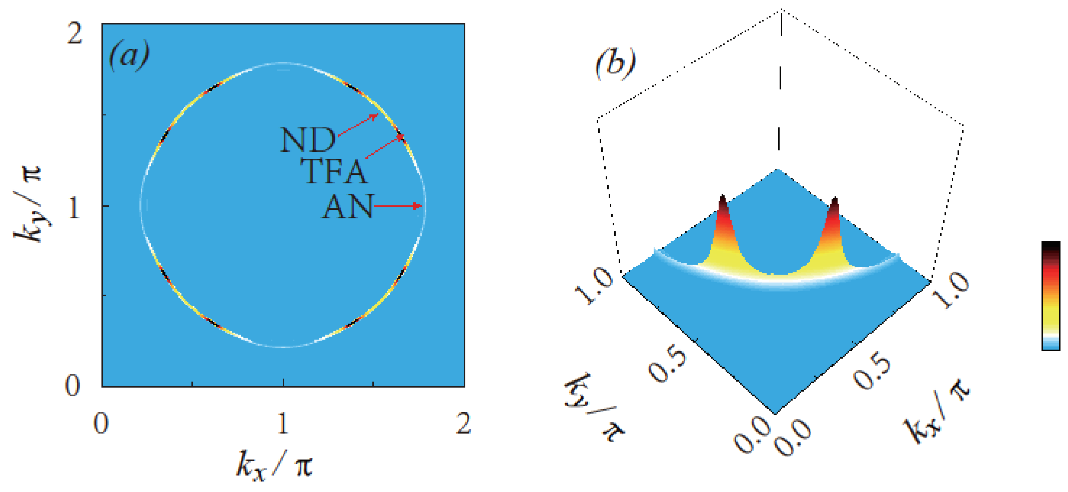
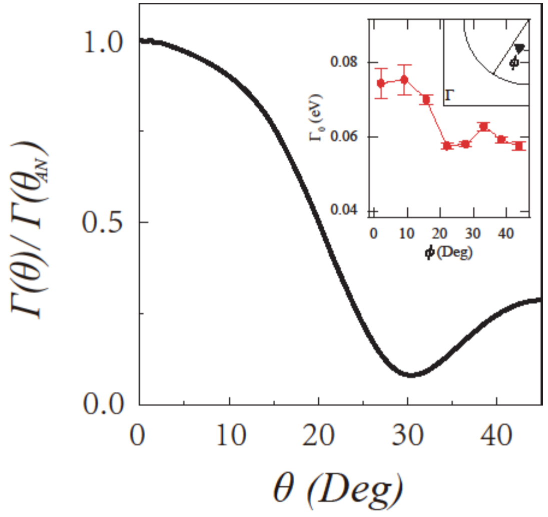
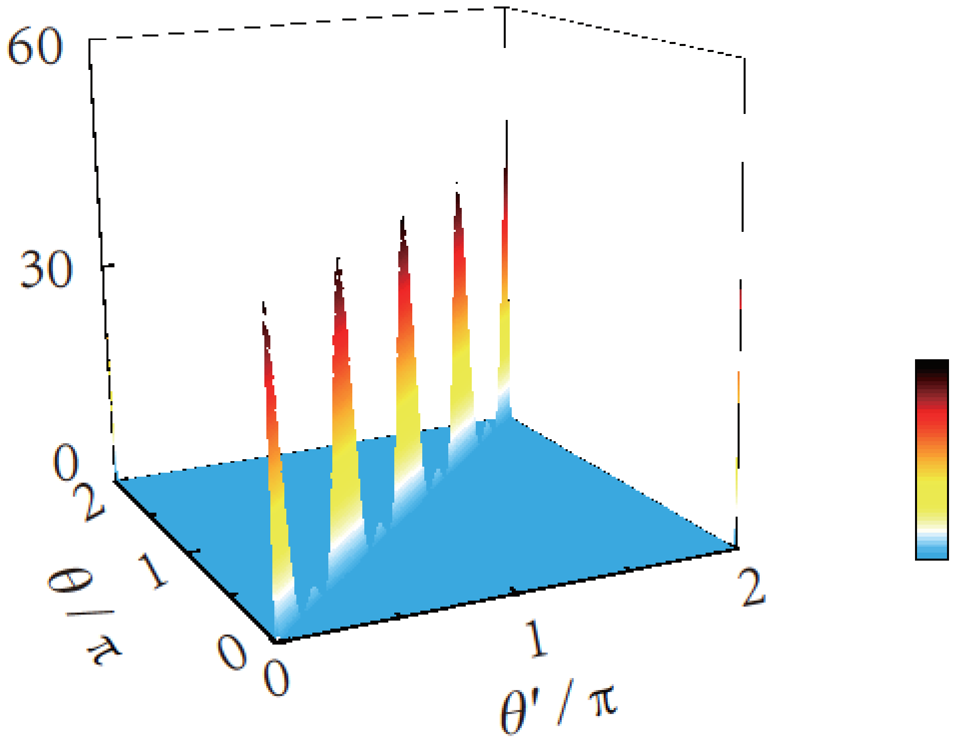


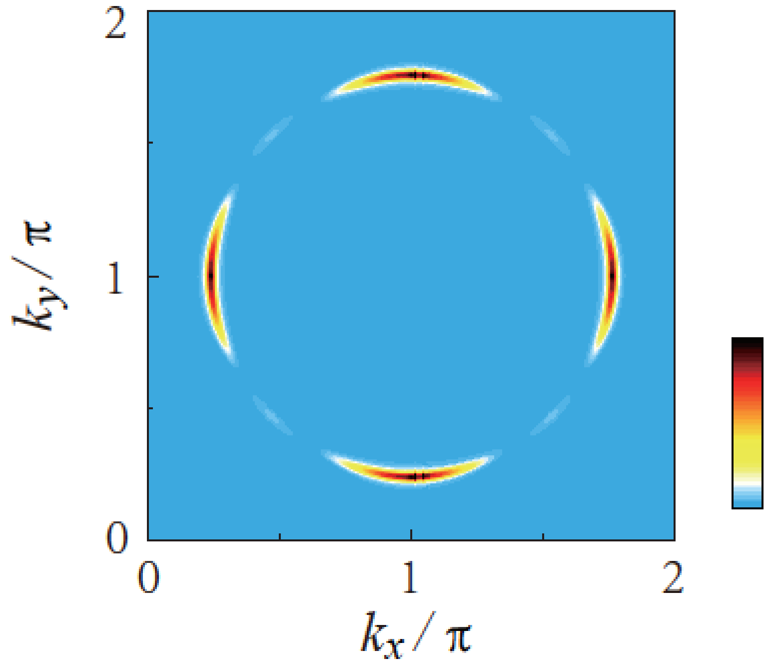
fig （a） is the weight of electronic quasiparticle spectrum of electron doping cuprate。fig （b） is the 3D distribution corresponding to (a) 。Then， δ= 0.19，T = 0.002J， and t/J = -2.5，t′/t = 0.3。



Scattering rate as a function of Fermi angle at δ= 0.19， T = 0.002J， and t/J =-2.5 ，t′/t = 0.3， inset is the scattering rate of $Pr_{1.3-x}La_{0.7}Ce_xCuO_4$[4]

Scattering kernel function and effective spin propagator





Scattering kernel function as a function of Fermi angle, where, AN marked the maximum at the antinodal region , ND marked another peak at the nodal region, TFA is the tips of Fermi arc., with t/J =-2.5, t′/t = 0.3, and T = 0.05J, δ= 0.19。

The distribution of Imaginary part of spin excitation propagator ImP(k,p−k,k′,ω) in (p_x ,p_y) plane, t/J = -2.5, t′/t = 0.3, and T = 0.04J δ= 0.19, where ω= 0.05J。

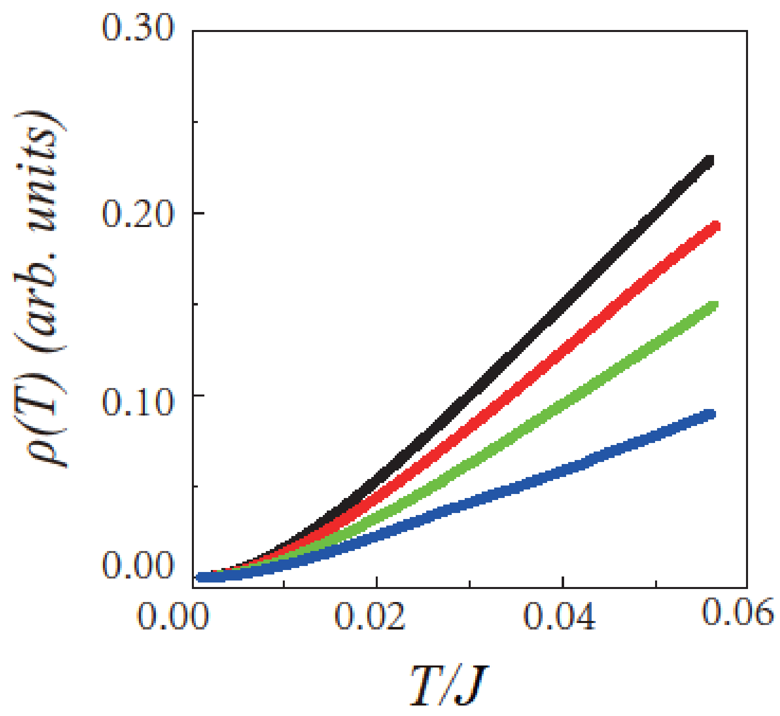
Current function： $$\mathbf{J} = en_0 \frac{1}{N} \sum_{\mathbf{k}} \mathbf{v}_{\mathbf{k}} \frac{dn_{\mathrm{F}}(\bar{\varepsilon}_{\mathbf{k}})}{d\bar{\varepsilon}_{\mathbf{k}}} \tilde{\Phi}(\mathbf{k})$$
$$= -en_0 \frac{\mathrm{k_F}}{\mathrm{v_F}} \int \frac{d\theta}{(2\pi)^2} \mathbf{v}_{\mathrm{F}}(\theta) \Phi(\theta).$$

Substituting the solution of the Boltzmann equation into the current function, we obtain the expression for the DC conductivity.：

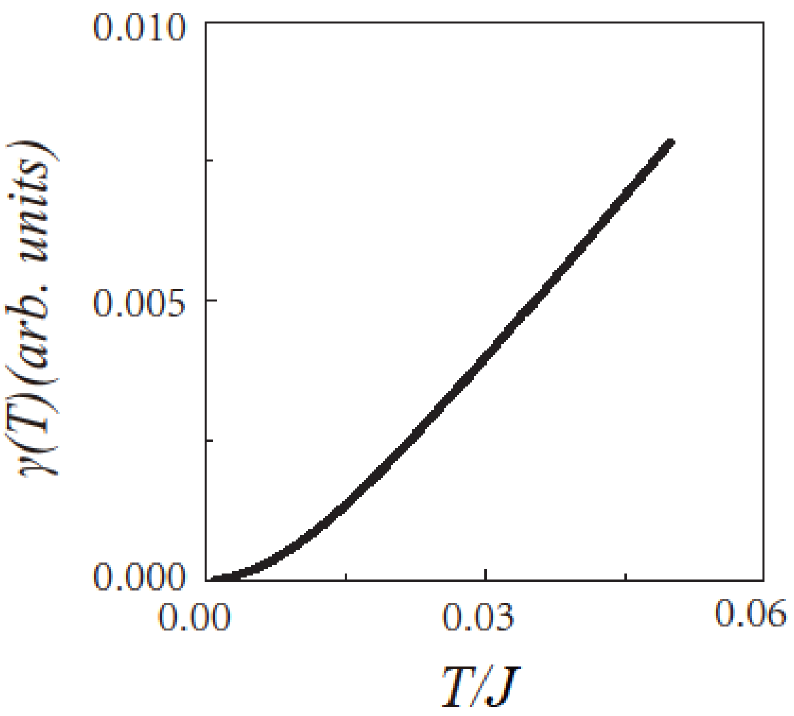
$$\sigma_{\mathrm{dc}}(T) = \frac{1}{2} e^2 n_0 \mathrm{k_F} \mathrm{v_F} \int \frac{d\theta}{(2\pi)^2} \cos^2(\theta) \frac{1}{\gamma(\theta)},$$

DC resistivity： $$\rho(T) = \frac{1}{\sigma_{\mathrm{dc}}(T)}.$$

T-linear resistivity and scattering rate



Resistivity as a function of temperature, t/J = -2.5, t′/t = 0.3, black line is δ=0.15, red line is δ=0.17, green line is δ= 0.19, blue line is δ=0.24 。



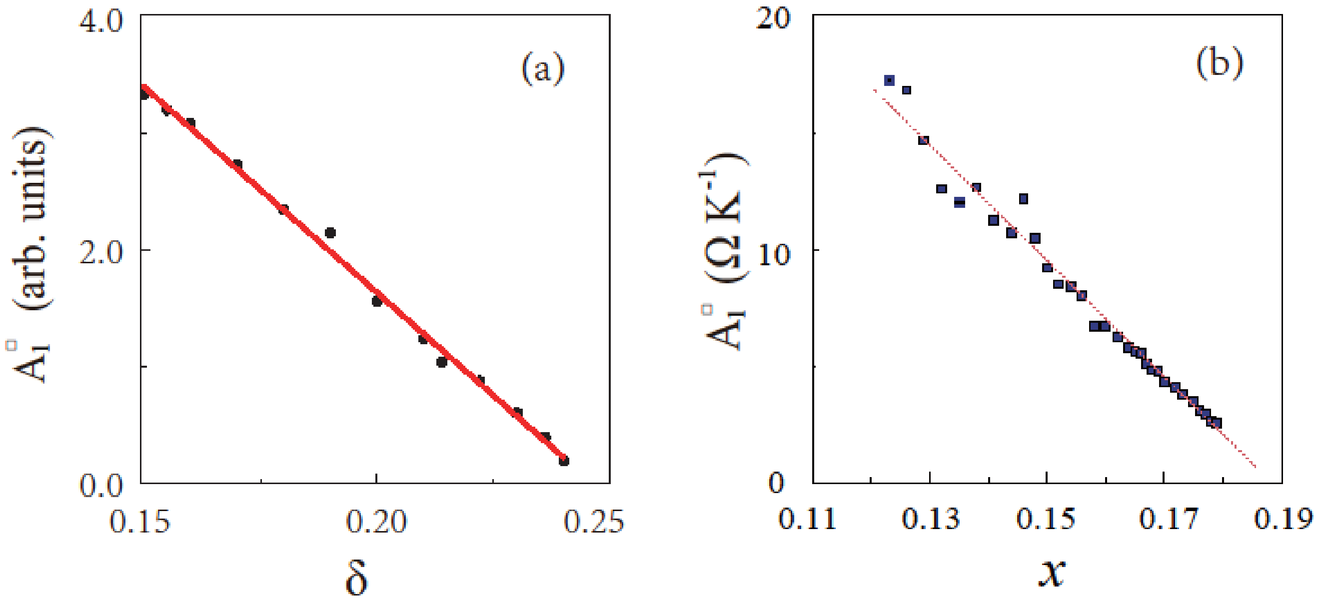
scattering rate on the antinodal region $\gamma(T)$ as a function of temperature with t/J = -2.5, t′/t = 0.3, and doping δ= 0.19 。

T-linear resistivity for different electron doped cuprate

The coefficient of dependence of low temperature linear resistivity on temperature is denoted as A_1, which decreases with the increase of doping concentration. This is consistent with the physical fact that the larger the doping concentration, the smaller the resistance. For comparison with experiments, we define

$$A_1^{\square} = A_1/d$$

where, d is the lattice length in the c-axis direction of electron doped cuprate high-temperature superconductor.



Coefficient of T-linear resistivity as a function of doping concentration, fig (a) is the result of our theory， fig (b) is the experimental result of $La_{2-x}Ce_xCuO_4$ [6]。

Superconducting transition temperature of electron doping cuprate superconductor：

The superconducting transition temperature of is obtained through the mean-field self-consistent method, with the condition being the superconducting gap Δ = 0. By simultaneously solving the mean-field self-consistent equations, the obtained temperature is the superconducting transition temperature T_c.

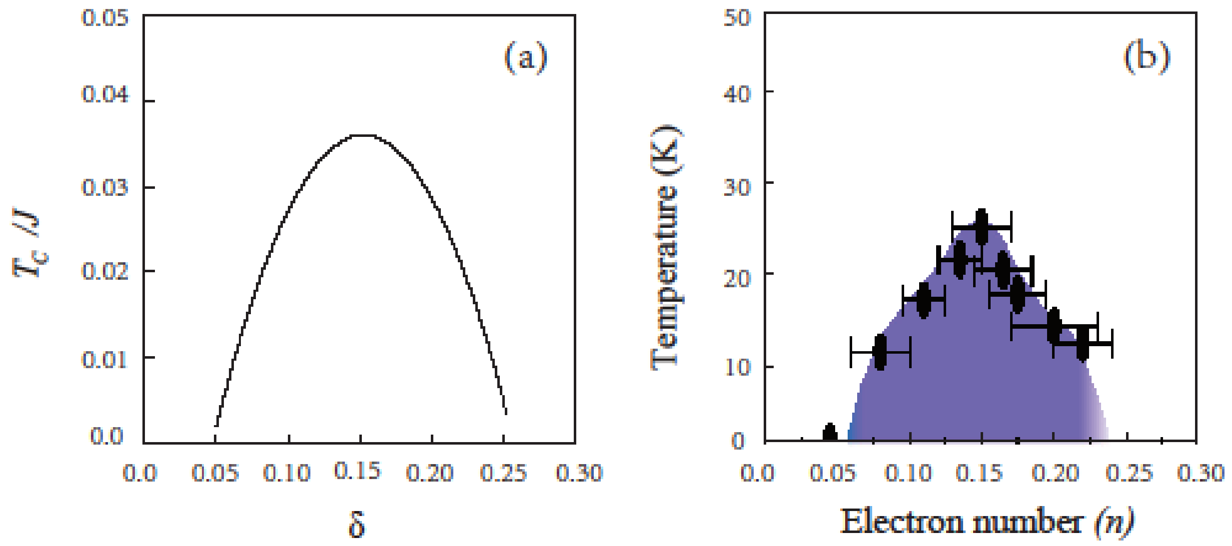
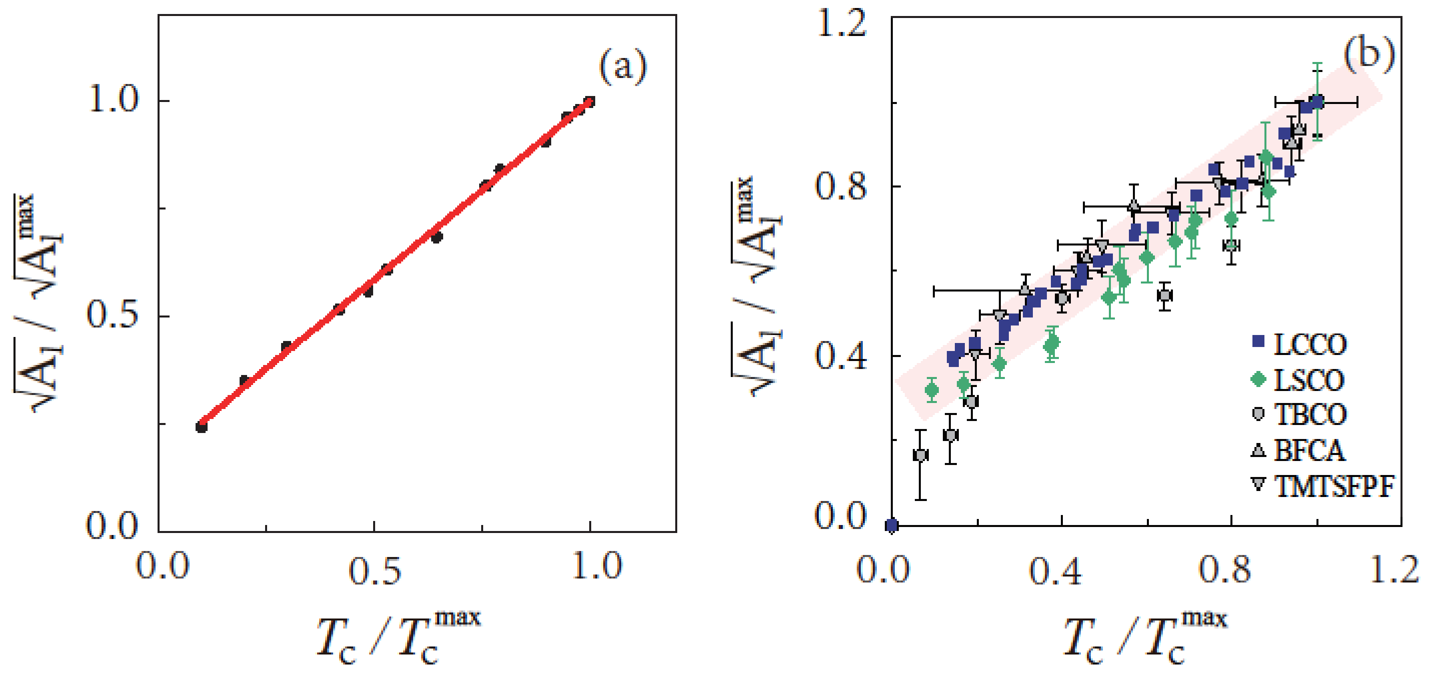


Fig (a) is the supeconducting transition tempereture T_c as a function of doping concentration, fig（b）is the experimental result of $Pr_{1.3-x}La_{0.7}Ce_xCuO_4$ [5]。

The relationship of the coefficient of T-linear resistivity and T_c:



the coefficient of T-linear resistivity as a function of superconducting transition temperature T_C, fig (a) is the result of our theory, fig (b) is the experimental result [6]。

Normal self energy and abnormal self energy：

$$
\begin{aligned}
\Sigma_{\mathrm{ph}}(\mathbf{k}, i\omega_m) &= \frac{1}{N^2}\sum_{\mathbf{k}_1,\mathbf{k}_2}\Lambda^2_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}}\frac{1}{\beta^2}\sum_{i\omega_{m1},i\omega_{m2}} G(\mathbf{k}+\mathbf{k}_1, i\omega_m + i\omega_{m1})D^{(0)}(\mathbf{k}_1, i\omega_{m1}) \\
&\quad \times D^{(0)}(\mathbf{k}_1+\mathbf{k}_2, i\omega_{m1}+i\omega_{m2}), \\
\Sigma_{\mathrm{pp}}(\mathbf{k}, i\omega_m) &= \frac{1}{N^2}\sum_{\mathbf{k}_1,\mathbf{k}_2}\Lambda^2_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}}\frac{1}{\beta^2}\sum_{i\omega_{m1},i\omega_{m2}} \Gamma^{\dagger}(-\mathbf{k}-\mathbf{k}_1, -i\omega_m - i\omega_{m1})D^{(0)}(\mathbf{k}_1, i\omega_{m1}) \\
&\quad \times D^{(0)}(\mathbf{k}_1+\mathbf{k}_2, i\omega_{m1}+i\omega_{m2}),
\end{aligned}
$$

conclusion：

The anomalous self-energy leads to electron pairing, which is caused by this spin propagator, resulting in an ultra-high superconducting transition temperature. The normal self-energy leads to electron scattering through the exchange of this spin propagator, resulting in T-linear resistivity. Therefore, the same effective spin propagator is responsible for both superconductivity and T-linear resistivity.

Summary :

1. T-linear resistivity is the nature result of the strong correlation electron system,

We just abstract an umklapp scattering process from normal self energy, and in this process the electrons exchange momentum thought exchanging an spin propagator, and the propagator is come from the kinetic energy driven superconductivity.

2. The superconductivity and the T-linear resistivity both are the results of the spin excitation, which drive the electrons paring in superconducting state, and dominate the umklapp scattering process. As an result, the T_C and coefficient of T-linear resistivity give a consistent result to the experiment.

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3. **Xingyu Ma**, Minghuan Zeng, Zhangkai Cao, and Shiping Feng, arXiv:2310.12414.
4. **Xingyu Ma**, Minghuan Zeng, Huaiming Guo and Shiping Feng, arXiv:2405.16778

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Thanks for your attention!